Extending the Fundamental Law of Investment Management

(Or How I Learned to Stop Worrying and Love Shorting Stocks*)
Abstract

• A theoretical framework which would allow for the decomposition of Information Ratio into its building blocks would be a powerful tool for managers and investors

• Such a framework was originally outlined by Grinold, then further refined by Clarke, de Silva, and Thorley (“CdeST”)

• By isolating each of the variables in the CdeST framework we can examine its sensitivity and impact on Information Ratio

• We explore the calculation of breadth when forecasts are correlated

• We expand the CdeST framework to include the dynamic of turnover and turnover cost

• We use the framework to illustrate the limited advantage of making more granular distinctions in return forecasts

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* with apologies to Stanley Kubrick for dragging Dr. Strangelove into the world of quantitative investment management
Information ratio, or the ratio of excess return to tracking error, measures the risk efficiency with which a manager delivers excess returns. High information ratios need not be seen as an alternative to high excess returns. Managers with higher information ratios simply deliver more active return for a given level of active risk.

As we have discussed in previous papers, investors are far more likely to prematurely (and ultimately incorrectly and expensively) terminate long-run skillful managers with lower information ratios than those with higher ones. But how do we identify those managers likely to have higher information ratios going forward, and how do managers adapt their investment processes to achieve more efficient delivery of excess returns?

Observing historical information ratios is a logical first step, but an approach that would allow us to estimate future information ratios (and deconstruct historical ones) based on more easily understood building blocks would be even more powerful. In a 1989 article entitled “The Fundamental Law of Active Management,” Richard Grinold proposed his well respected framework for predicting the ex-ante information ratio of a manager based on two variables — skill and breadth. He measured skill as the information coefficient (IC), or correlation between the manager’s expected excess returns and the subsequent excess returns of the assets, and breadth as the number of independent forecasts in the portfolio (N).

\[ IR = IC \times \sqrt{N} \]

**EFFICIENCY = SKILL * BREADTH**

Where

IR = Information Ratio

IC = Information Coefficient

N = Number of Independent Forecasts

The IC for the perfectly brilliant manager is 1, and the perfectly wrong manager (just as valuable a resource as long as you know of his “skill” in advance!) is -1. A manager whose insights were random with no skill would have an IC of 0. Equally impactful was the second term in the equation. The greater the breadth of insights, the greater the efficiency of the portfolio — an intuitive relationship.
In 2002, Roger Clarke, Harindra de Silva, and Steven Thorley (“CdeST”) powerfully extended Grinold’s framework in a paper entitled “Portfolio Constraints and the Fundamental Law of Active Management.” The authors added a third term which we can describe as freedom to implement. They called it the Transfer Coefficient, and defined it as the correlation of forecast returns and bets in the portfolio.

\[ \text{IR} = \text{IC} \times \sqrt[\frac{1}{2}]{\text{N}} \times \text{TC} \]

EFFICIENCY = SKILL * BREADTH * IMPLEMENTATION

Where

TC = Transfer Coefficient

This additional term relaxed a rather simple assumption in Grinold’s model, that managers face no constraints which preclude them from translating their investment insights directly and freely into portfolio bets. In the CdeST model, the more closely the portfolio bets are aligned with the manager’s predicted returns for each stock, the higher the TC. A manager with no constraints could theoretically achieve a TC as high as 1 (resulting in the same predicted outcome as Grinold’s framework). To the extent that a manager faced constraints, her information ratio would be lower than the Grinold framework predicted. A manager with constraints so binding that they kept her from making any bets that reflected her insights would have a TC of zero.

**Understanding the Variables**

We can bring the CdeST equation to life for the practitioner if we isolate each of the variables and examine the scale and nature of its impact on theoretical information ratio. For those readers who remain unimpressed by an increase in information ratio, feel free to think of the impact as a change in excess return for a given level of tracking error.

**INFORMATION COEFFICIENT**

\[ \text{IR} = \text{IC} \times \sqrt[\frac{1}{2}]{\text{N}} \times \text{TC} \]

As anyone who has managed portfolios would surely report, correctly predicting the general order, no less the level, of subsequent returns for a group of securities or asset classes is a tall order. Success is measured in small victories, not perfection, over long
periods of time. In practice, managers with a broad set of investment decisions can achieve significant risk adjusted excess returns with information coefficients between 0.05 and 0.15.

Exhibit One: Resulting information ratios for varying levels of information coefficients

Exhibit One represents a manager with a breadth of 500 using three different Transfer Coefficients (1.0, 0.5 and 0.3). We can see that quite attractive theoretical information ratios can be indicated for relatively small information coefficients, before accounting for turnover constraints and transaction costs.

Choosing the "correct" calculation for the realized return improves the explanatory power of the framework. A manager who invests on a sector neutral basis would be most effectively analyzed by calculating the correlation of the predicted returns of each stock to the subsequent excess return of that stock versus its sector. A manager who was not sector neutral might calculate the correlation of predicted returns of stocks and either the subsequent returns of each stock, or the returns versus the benchmark — both will result in the same correlation of predicted and realized returns. If we calculate ICs for a sector neutral investor using realized excess returns versus the market instead of the sector, the IC would contain additional noise which would make the evaluation framework less reliable. While we use this framework to separate skill from implementation, the more relevant the measure of skill to the ultimate portfolio implementation, the more insightful the output of the framework. In Appendix One of this paper, we discuss more fully the implications of using various degrees of granularity in forecasted return.
Breadth

\[ IR = IC \times \sqrt[N]{N} \times TC \]

In this framework, expanding the universe of independent forecasts will increase breadth and, all other things being equal, information ratios by the square root of the increase in number of assets in the forecast pool. However, only those names for which the long-only manager actively forecasts returns will improve his breadth. Moving to a broader benchmark with more securities for which the manager does not forecast returns will increase the number of unintentional underweights, but not breadth. In fact, the increase in unintentional bets (not owning names in the benchmark for which the manager has no return forecasts) ultimately reduces TC, which reduces IR.

![Exhibit Two: Resulting information ratios for varying levels of breadth](image)

In Exhibit Two, we can see that breadth has a positive impact on IR, with the greatest improvement coming as \( N \) moves from 1 to 100. All managers should gain efficiency by expanding their number of forecasts, but this framework clearly identifies that the incremental efficiency gain of expanding the number of independent forecasts from 50 to 150 (\( \sqrt{150} / \sqrt{50} -1 \), or 73%) is far greater than that by extending from 500 to 600 (\( \sqrt{600} / \sqrt{500} -1 \), or 10%).

Expanding breadth will have a greater marginal impact on information ratios for higher IC and TC managers. Additionally, if the cost of expanding breadth is a lower IC on either the existing set of forecasts (e.g., spreading the analysts’ time more thinly) or the additional forecasts (e.g., venturing into forecasts for which the analyst has less insight), the marginal gains from a wider universe may be illusory.
Ambiguities arise from two issues: expressing breadth in the same annualized terms as the other variables in the equation, and the complication that increasing the number of assets under research coverage may not proportionally increase the effective number of independent decisions.

We can think of the Grinold or CdeST equations either in annualized or periodic terms, but we must be consistent. IC and TC have no concept of time, as both are correlations between two sets of data for a single point in time, though we might typically use the mean of a time series of those correlations to smooth our results. Since information ratio is by definition the annualized excess return divided by the annualized tracking error, \( N \) must be the number of independent active forecasts per year.

We think of breadth as having two dimensions — one cross-sectional dimension (number of potential assets at any point in time) and one time-series dimension (how many times you re-evaluate each asset per year). If at each monthly decision point the manager truly had independent forecasts for each of his 500 assets and if the return forecasts for each of the twelve months of the year were also truly independent, he would have a theoretical \( N \) of 6,000. However, we recognize that there can be considerable correlation in both dimensions which can result in actual \( N \)'s that are substantially lower.

Take for example a consumer cyclical equity analyst who is bearish on the auto industry. By electing to underweight three auto original equipment manufacturers (OEMs), is he truly expressing three independent views, or is his breadth actually less than three? Are active positions based on thematic ideas or common factors truly independent? To the extent the cross-sectional forecasts are correlated, \( N \) will be overstated.

Correlation plays a similar role in the time-series dimension. Especially for investors whose forecast horizons are either intermediate in nature, or are derived from slow moving variables, there can be substantial correlation between the forecasted return for (or ranking of) each asset from period to period. As in the cross-sectional component, to the extent the times-series forecasts are correlated, \( N \) will be overstated.

We are continuing to work on the estimation of a multiplier for \( N \) to reduce \( N \) appropriately for the correlations in each of these two dimensions. Early work in
the large cap equity space for intermediate term forecasts and monthly re-forecasting would indicate that it might reduce the calculated N (number of stocks times number of observations per year) for a monthly decision frequency by 50-75%. Thus, a manager with 500 assets under research coverage re-evaluating their portfolio 12 times a year might have an adjusted N of 2,400 rather than 6,000. The degree of the adjustment factor is highly dependent on the degree of correlation of the manager’s signal both cross-sectionally and over time.

As a side note, we observe that market participants with a small number of assets to choose from and low transaction costs (for example, government bond traders) often increase their turnover rates, effectively increasing breadth by shortening holding periods. By doing so, they can achieve the diversification powers of breadth in a world which does not offer them a wide range of asset choices.

However, this trick only works for investors with low transaction costs and low serial correlations of forecasted returns. Using our extended equation which includes transaction costs later in this paper, we can see that unless transaction costs are low enough, increasing turnover and N proportionally results in a deterioration of information ratio. Similarly, investors with high serial correlation of forecast returns who attempt to increase N by re-evaluating more frequently will find that these theoretical gains are elusive, as the “true” adjusted N does not necessarily increase.

**Transfer Coefficient**

\[ IR = IC \times \sqrt{N \times TC} \]

The transfer coefficient can be thought of as the efficiency with which the manager translates her insights into her portfolio bets. It is one of the most effective levers a portfolio manager can use to increase information ratio.

In Exhibit Three, we can see that using a non-linear optimizer to construct long-only portfolios with different levels of tracking errors, the more concentrated the portfolio and the greater number of unintentional bets the portfolio takes on, the lower the initial setup Transfer Coefficient. This is an important driver in understanding why concentrated, high-tracking error active portfolios have lower information ratios, and the potent argument for enhanced indexation. While it might make intuitive sense that making large bets on a small number of the manager’s highest
return expectation stocks might increase Transfer Coefficient, in fact, the highly concentrated market capitalization of a benchmark such as the S&P 500 produces increasingly greater inefficiencies in portfolio construction as positions are concentrated. These issues are discussed in a previous piece we have published, “Reach for More Return — But Don’t Hurt Yourself.”

Since most investors have a need to reach for higher levels of excess return, and since it would be unreasonable to believe that these higher returns would not be accompanied by higher levels of tracking error, how does an investor increase risk and return, but not suffer deterioration in the transfer coefficient? The answer lies in relaxing the constraint on selling stocks short.

Note: Theoretical optimized portfolios. Results will vary for different alphas, benchmarks, risk parameters and optimization engines.
In Exhibit Four, we see the results of using a non-linear optimizer to construct optimized portfolios with different levels of relaxation of the short sale constraint at two different levels of tracking error — 2% and 6%. The long-only portfolios are fully invested portfolios with no other constraints optimized versus the S&P 500 (and are the same as the long-only portfolios at these risk levels in Exhibit Three). The “110/10” portfolios are 110% of invested capital long, 10% of invested capital short, optimized versus the S&P 500. Each of the 120/20 through 150/50 is the same, with higher degrees of gross exposure, but a net of 100% market exposure, and the same tracking error. The 100/100 portfolios are 100% of invested capital long, 100% of invested capital short, and the targeted tracking error of long versus short. This portfolio is a hypothetical market neutral portfolio which could then be equitized with S&P 500 futures, resulting in the same tracking error to the S&P 500.

We can draw two primary conclusions from this example.

- Relaxing the short sale constraint has a significant positive impact on transfer coefficients and, by extension, the level of excess return managers are expected to achieve for a given level of active risk.

- More than half of the gap in TC between the long-only and the 100/100 portfolio is closed in the first 20% of shorting, and approximately two-thirds with 30%. This is consistent with the new wave of hybrid equity products that are seeking more efficient ways to achieve higher levels of return with profiles of 130% long and 30% short.
What causes the transfer coefficient to rise as we relax the short sale constraint? Observing the dynamics in a single sector of an equity portfolio can help us understand what leads to higher returns for the same level of risk when shorting is allowed.

Imagine a manager with a particular set of information about a sector with 4% of the capitalization of the manager’s benchmark. In Exhibit Six, with a short sale constraint in force, the manager invests the entire 4% in the name which has his highest expected return. While this might make sense at first, making a large 3.75% positive bet in his most attractive name, in a long-only environment this also leads to many unintentional bets.

Stocks B and C, which the manager also expects to outperform, have negative bets versus the benchmark. Stocks F and G, which the manager expects to significantly underperform, have only small negative bets, limited to their capitalization weight.

Once the short sale constraint is lifted, however, the manager can now scale his bets much more closely to his insights. This results in making the same total amount of bets as the long-only portfolio, 7.50%. However, if the manager’s forecasted returns are realized, the excess return from the long-only portfolio would be only 0.74%, and the long/short portfolio 1.25%. Thus we can see the effect of the higher transfer coefficient that results from the lifting of the short-sale constraint, and how it can enable a portfolio to achieve higher returns for the same level of risk and insight.

<table>
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<th>Stock</th>
<th>Expected Return</th>
<th>Benchmark</th>
<th>Long-Only Portfolio</th>
<th>Long-Only Bet</th>
<th>Long/Short Portfolio</th>
<th>Long/Short Bet</th>
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<tr>
<td>A</td>
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<td>4.00%</td>
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<td>0.00%</td>
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<tr>
<td>C</td>
<td>5%</td>
<td>1.00%</td>
<td>0.00%</td>
<td>-1.00%</td>
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<td>E</td>
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<td>F</td>
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<td>G</td>
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Total Bets (proxy for risk) 7.50% 7.50%

Active return if expected returns realized 0.74% 1.25%
We explore the advantages of removing the short-sale constraint in our Equity Insights piece entitled “Why you should let your ‘long-only’ manager short stocks.” While these simple portfolio construction rules illustrate the sensitivities of the transfer coefficient to certain key variables, they do not begin to reflect the more complex world of a live portfolio. Two primary pieces are missing: portfolio turnover, and the transaction costs which accompany that turnover.

**A Missing Component: Ongoing Turnover**

As powerful a framework as Clarke and his co-authors presented, they chose to address portfolio turnover in a rather basic way — allowing for only one portfolio rebalancing from a starting index portfolio, and examining the outcomes allowing for different levels of turnover. Unfortunately, this approach misses two important dynamics of real world portfolio management. First, the world keeps changing — prices change, alpha signals change — and as a result, portfolios need to change if the subsequent information is to be incorporated and exploited. Second, trading the portfolio is not free — the interaction of these two factors is significantly entwined with the IR framework. Managers seek to reduce expense by minimizing turnover, but as turnover is constrained, the portfolio bets drift away from the pure alpha insights, leading to lower transfer coefficients.

We have attempted to extend the CdeST framework by adding two additional components — multi-period turnover and transaction costs. We hope that observing more practically derived transfer coefficients will allow the framework to help attribute the historical performance of existing strategies and more realistically examine the future prospects of both seasoned and as yet untried strategies.

We know that for a given transaction cost per dollar of stock traded (TCost) and turnover rate (TR) we can calculate the performance impact of turnover on the portfolio. Further, if we know the tracking error of the portfolio ex-ante, we can convert this measure from return terms to information ratio terms, and include it in our calculation.

By industry convention, a 100% annual turnover signifies that a portfolio with a market value of $100 buys $100 in stock and sells $100 in stock per year. For each $1 of turnover we both buy and sell, incurring two times the TCost. We can represent the annual return impact as:
Annual Return Impact = TCost * TR * 2

Converting this from return terms to information ratio terms, and substituting back into the formula represented by CdeST, we now can include transaction costs in our model:

\[ IR = IC \times \sqrt[N]{\text{TC} - (\text{TCost} \times \text{TR} \times 2/\text{TE})} \]

Where
- TCost = Average market impact per transaction in % of market value
- TR = Annual Portfolio Turnover Rate
- TE = Portfolio Tracking Error
- TC = Transfer Coefficient

Let’s look first at the more straightforward impact of turnover — the impact of transaction costs. For the sake of simplicity, we will initially assume that a change in turnover has no effect on transfer coefficient. Later we will relax that assumption.

In Exhibit Seven, we can see that per trade transaction costs increase, the incremental impact on information ratios is significant. This impact is accentuated for higher turnover strategies (e.g., the 200% turnover strategy above) but is still significant even for a 50% per year turnover strategy.
With just this information in hand, the reader might jump to the conclusion that trading less should lead to higher information ratios. Unfortunately, it isn’t quite that simple. While indeed transaction cost impact declines as turnover is decreased, the Transfer Coefficient also falls as turnover is decreased, reducing the information ratio. This occurs because a portfolio which turns over less is always further away from its target, based on the manager’s investment signal. This effect multiplies, as turnover constraints move the portfolio further from its target over time, with the degree of deceleration dependent on the interaction between the degree of the constraint and the level of risk in the portfolio.

Exhibit Eight above illustrates the effect of turnover constraint on the Transfer Coefficient in a long-only portfolio. In this example, we compared two theoretical portfolios at three risk levels. For each of the three risk levels, two identical optimal starting portfolios were created on “Month 1.” Then both portfolios were rebalanced each month, one with a methodology designed to achieve specified turnover rates for each risk level (35%, 50%, and 100%), and one without turnover constraints — and compared. The pattern is clear — a new setup carries with it a TC that fully reflects the investment insights of the manager. But as time passes, in order to control transaction costs, the less than full turnover realized by the portfolio causes the TC to drift lower, losing approximately 0.08 to 0.10 in TC over the first 18-24 months in the particular portfolios we modeled, then stabilizing (but not recovering). This highlights the risk of overestimation of TCs when “new setup” data is used (as the CdeST paper did) in evaluating TCs. We have not yet derived a more general expression to describe this drift in TCs, but would expect the magnitude and path of the decline to vary across portfolios.
We would equate the Month 1 Transfer Coefficient to the CdeST TC variable, and the Month 20 Transfer Coefficient as our TC’. With this new “seasoned” TC’ we can restate the equation as:

\[ IR = IC \times \sqrt{N \times TC'} \times (TCost \times TR \times 2/TE) \]

Where

TC’ = Steady State Transfer Coefficient after accounting for turnover constraints

Based on these two variables and the coincident turnover rates that drive them, if the informational decay function of a particular signal could be estimated, one might be able to derive a turnover rate which maximizes information ratio. We will reserve this question for future examination.

**Conclusion**

The analytical framework originally created by Grinold and later extended by Clarke, et al., is a powerful and flexible construct for understanding and improving manager risk adjusted performance. By adding transaction costs and the impact on Transfer Coefficient of constraining turnover, the framework becomes even more realistic, and thereby more applicable to actual manager strategies. It clearly illustrates that strategies which allow for greater breadth by expanding the number of uncorrelated forecasts increase efficiency, especially in the earlier stages of expansion. It also explains that transfer coefficients, and therefore information ratios, are highest in portfolios which allow shorting, but with long-only portfolios are higher in lower tracking error portfolios such as enhanced index. Armed with these facts and a simple set of equations for estimating their impact on performance, managers and investors now have an additional robust language for discussing alternatives for improving performance and setting effective targets for risk and the degree of portfolio constraints.
Appendix One: Granularity in Alpha Signals and the implications for portfolio efficiency

Measuring ICs can also help us understand how fine-tuned our return forecasts need to be to achieve investment success. Imagine two managers whose processes each produce forecasted returns for 500 stocks. Manager One is good, with an IC of 0.05. Manager Two is very good, with an IC of 0.10. While each manager’s research department computes expected returns for each stock down to two decimal places, they prefer some PMs to others. In an attempt to favor their preferred PMs, each provides their forecasts to their five portfolio managers with different levels of granularity. Their favorite PM gets the actual expected return, and in descending order of preference the other PMs get four descending granularities of forecasted return — rank order, decile rank (1-10), quintile rank (1-5), and tertile rank (1-3).

Most investors’ first instinct is that telling a portfolio manager the expected return of a stock down to two decimal places must yield a significantly higher information ratio than “blurring” the data by only telling the PM into which third of the universe a stock’s expected return falls. In fact, the deterioration is less than one might expect.

To measure the effect of granularity we generated normally distributed random “alphas” and “ex-post returns,” biasing the random returns such as to produce “managers” for whom “good” average ICs (0.05) and very good average ICs (0.10) were “achieved.” We converted the normally distributed random forecasted returns to ranks, deciles, quintiles and tertiles, and then simultaneously computed information coefficients for all five measures versus the single series of ex-post return random returns.

Decreasing the granularity of information from actual forecasted returns to the far less granular quintile ranking based on those forecasted returns decreased the IC by only 6% for both managers. This equates to a decrease in excess return of 0.12% for a manager delivering 2% excess returns. Decreasing the granularity to tertiles (equal numbers of stocks you like, hate, and don’t care about), the drop in IC is larger, but still relatively modest, at 11%, equating to 0.22% excess return for our 2% excess return manager.
The modest drop-off reinforces the power of “getting the sign right” in investment management and highlights the value of screening techniques and the power of broadly successful insights. Of course, the Information Coefficient is only one of the terms in our equation. However, our work has indicated that Transfer Coefficients remain relatively constant as granularity decreases. If breadth, turnover, and transaction costs were to remain constant, the resulting information ratio of the strategy would only decline modestly as the forecasted returns became more granular.
Appendix Two: Quantifying qualitative insights

One challenge to using the structure is that it does not capture the value added from qualitative insights that may be incremental (or detrimental) to the primary measured alpha source. The fundamental law assumes that an investor’s “skill” is entirely captured in his alpha signal. However, a portfolio manager may make mental adjustments to his forecasts based on additional information which cannot be quantified. "Overriding" these forecasts may enhance, or detract from the investor’s true IC, but regardless will not be captured in the general calculation. In a perfect world, we would ask the portfolio manager to record his “adjusted” signal, and we could measure his adjusted IC directly. However, in practice this is not likely to occur. Rather, we are reduced to adjusting the primary IC by some factor which we think explains the value added (or detracted) by the PM adjustments.

An example of this is the portfolio manager, who despite using an optimizer, overrides a recommended concentrated position in a stock with an outlier alpha knowing that extreme alphas have historically been poor predictors of future performance. If the PM’s insight proves correct, his true IC will be higher than his computed IC because the adjusted lower forecast return will be more highly correlated with experienced returns.

In addition, the investor’s TC will also be impacted by this because he constructs his portfolio based on “enhanced” forecasts, while the TC is naively measured against unadjusted ones. For a portfolio manager who adds value through his enhanced forecasts or subjective risk management, and whose bets align more consistently with these enhanced forecasts than the unadjusted ones, the observer would need to increase both the IC and TC by some reasonable factor in order to bend the framework to appropriately adjust for these overrides.
Footnotes


Additional References


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