lifeMetrics℠

A toolkit for measuring and managing longevity and mortality risks

Technical Document

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Acknowledgements

The development of the LifeMetrics toolkit has been a team effort which has benefited from the input of many clients, advisors and colleagues, all of whom have helped us to create this platform.

In particular, we acknowledge the contributions of our LifeMetrics Advisors, Watson Wyatt and the Pensions Institute at Cass Business School.

We thank Robert Hall and Daniel Ryan from Watson Wyatt in the UK, together with Jesse Schwartz and Simon Poon from Watson Wyatt in New York for their assistance and advice in development of the LifeMetrics Index and LifeMetrics Framework.

We are also grateful to Prof. David Blake, Director of the Pensions Institute, for his contribution as a co-founder of the LifeMetrics Index. Additionally, we thank Prof. Blake, together with Prof. Andrew Cairns, of Heriot-Watt University, and Prof. Kevin Dowd, of the University of Nottingham, for their role in leading a collaborative investigation into stochastic mortality models, the results of which are published in a working paper available on the LifeMetrics website.

The LifeMetrics platform has also benefited from discussions and meetings with many experts in the field including Prof. Jay Olshansky of the University of Illinois at Chicago, Stephen Richards of Richards Consulting in Edinburgh, Jay Shah of Synesis Life in London, Dave Grimshaw and Rajeev Shah from Barnett Waddingham LLP in London, Michael Hyman and Jacob Siegel, formerly of the US Census Bureau.

We also acknowledge the helpful assistance and advice of many individuals involved in the collection, analysis and publication of mortality and population data in both the UK and the US. This includes the UK Office for National Statistics and the Government Actuary’s Department, and in the US, the Centers for Disease Control and Prevention and the US Census Bureau. All the data currently used in the production of the LifeMetrics Index originates from these organizations.
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Part I
Primer
1. Introduction

1.1 Background

Longevity is a subject that has received considerable press coverage over the past twelve months. It has become an increasingly high-profile risk for defined benefit pension plans and a considerable concern for plan sponsors, who must ultimately meet the cost of increasing life expectancy. As sponsors update the mortality tables used to calculate their pension liabilities, many are experiencing a significant increase in the value of liabilities and, as a result, a substantial widening of their pension deficits.

A recent study of the companies in the UK’s FTSE100 index found that the assumptions about mortality rates and longevity used in pension valuations were overly optimistic, to the extent that realistic longevity assumptions would more than double the aggregate deficit from £46 billion to £100 billion (Pension Capital Strategies and Jardine Lloyd Thompson, 2006). On average, each additional year of life adds approximately 3-4% to the value of UK pension liabilities.

In the US, the Internal Revenue Service (IRS) has recently mandated new mortality assumptions to be used to set pension contributions, which according to Watson Wyatt will increase pension liabilities by 5-10% (Halonen, 2007). Similarly, Mercer Human Resource Consulting has calculated that the use of up-to-date mortality tables would increase the cost of providing a pension to a male born in 1950 by 8% (Mercer, 2006).

Meanwhile, many life insurers globally have become concerned about their exposure to catastrophic mortality events. This has resulted in the issuance of several bonds transferring mortality catastrophe risk to investors.

Collectively, longevity and mortality risk has historically been a very opaque risk. Until very recently it was largely ignored by pension plans, their sponsors and finance professionals in general. For a long time, only actuaries, demographers and insurance companies showed any interest in measuring and managing this risk. There were many reasons for this, including that it is a non-financial risk and unfamiliar to finance professionals; it has been measured and analyzed in a different way from financial risks; and, its impact on pension liabilities was virtually invisible to pension plan sponsors.

Most of the world’s longevity exposure resides not with insurers, but with corporations and public sector organizations as sponsors of employee benefit plans. Unfortunately, these sponsors are ill equipped to manage longevity risk. This unseen risk has been slowly eroding shareholder and taxpayer value. It is only with the recent changes to pension regulation and accounting standards in many countries (including the US and Europe) that longevity risk has become visible to corporate sponsors and their shareholders. But they still lack the tools to measure and manage it.

1.2 LifeMetrics

LifeMetrics is a toolkit for measuring and managing longevity and mortality risk, designed for pension plans, their sponsors, insurers, reinsurers and investors. LifeMetrics enables these risks to be measured in a standardized manner, aggregated across different sources and transferred to other parties. It also provides a means to evaluate the effectiveness of longevity/mortality hedging strategies and the size of any residual risk.

This toolkit comprises three components:

- LifeMetrics Index: Data for evaluating current and historical levels of mortality and longevity
- LifeMetrics Framework: A set of tools, methods and algorithms for measuring and managing longevity and mortality risk
- LifeMetrics Software: Software for forecasting future mortality rates

LifeMetrics has been designed to raise the profile of longevity and mortality risk management, increase the transparency surrounding this risk and provide practical tools and methods to measure and manage it. In particular, our goals in launching LifeMetrics are to:

- Assist pension funds and their sponsors in implementing more effective management of longevity risk
- Educate investors to promote development of a liquid market for longevity-linked and mortality-linked assets
• Provide alternative risk transfer solutions for insurers and reinsurers to complement their existing toolkits for both managing and sourcing longevity and mortality risk.

It is our hope that LifeMetrics will help to catalyze the development of a liquid market in traded mortality and longevity, to the benefit of the pension industry, the insurance industry and investors globally.

LifeMetrics has a number of differentiating features, including:

• International indices: Initially we are publishing index data for the US and England & Wales based on national population data. We plan to add other countries in the coming months.
• Transparency: All methodology, algorithms and calculations are fully disclosed and open.
• Standardization: LifeMetrics is explicitly designed to facilitate the structuring of longevity securities and derivatives. JPMorgan is developing flexible, standardized hedging instruments based on this index.
• Risk management: Tools to assist in developing forecasts of future longevity, analyze the impact on portfolios and design risk management solutions. LifeMetrics addresses the basis risk between actual populations at risk and the national population.

1.3 Mortality risk and longevity risk

Defined benefit pension plans, corporations, governments, insurers, reinsurers, long-term healthcare providers and individuals are all exposed to the uncertainties connected with how long people live. Although some of these uncertainties can be diversified by aggregating individual lives into large groups, they cannot be eliminated completely and can have significant economic consequences. Collectively these uncertainties are generally referred to as longevity risk, or mortality risk (depending on the context), and have become an important concern, most recently for defined benefit pension plans and their stakeholders.

Although the terms “longevity risk” and “mortality risk” are often used interchangeably, they are actually opposite sides of the same coin. Whereas longevity relates to the length of life, mortality relates to the rate of death. So, longevity risk describes the risk that an individual, or group of individuals, will live longer than expected, while mortality risk is generally used to describe the risk that an individual, or group of individuals, will live, in aggregate, shorter than expected (i.e., their mortality will be higher than expected).

In this document we will often use the term mortality to collectively refer to both mortality and longevity.

1.4 The study of mortality

The study of mortality relates to the survival and death of individuals within a particular population. It falls within the disciplines of demography and actuarial science, but is increasingly embracing biology, sociology, medicine and finance. Broadly speaking, mortality can be analyzed from two perspectives: statistical or biological. The statistical approach looks at what has happened in the past and extrapolates this into the future. The biological approach takes account of the health, medical condition, environment and lifestyle of the population and the likely impact of medical treatments available to them. Regardless of the method used, the understanding of mortality is integral to the management of pension funds and insurance companies.

Within a population one can analyze the probability of an individual surviving or dying over a given period. The probability of dying over a year is often referred to as the mortality rate for the individual. Because an individual can only either be alive or dead, the probability of survival over the next year is equal to (1 - mortality rate).

Mortality rates generally increase with age. For biological reasons, a young person is more likely to survive another year compared to an old person. However the relationship between age and mortality rate is not always monotonic. For example infants have higher mortality rates than young children; and twenty year olds may have a higher mortality rate than thirty year olds due to the greater number of fatal accidents experienced by young adults.

Historical data over long periods of time in many developed countries indicate that mortality rates for a given age have generally fallen with time. So a sixty year old today is more...
likely to survive another year compared to a sixty year old in 1950. This mortality improvement has mainly been due to better nutrition, healthier lifestyles and medical advances. However the relationship between time and mortality rate is not linear. In the future it is entirely possible that mortality might stop improving because of new diseases, increasing obesity and/or biological/medical limitations.

Other factors that affect mortality include gender (women tend to live longer than men), socio-economic group (the well-off tend to live longer than the less well-off), geography (there are differences between say Japan and the US, and even within countries), lifestyle (overweight smokers tend to have poorer mortality) and year of birth. In the UK, for example, cohorts (i.e., individuals within the population who were born in the same year), born in the 1930s have been observed to exhibit higher rates of improvement in mortality than those born in the 1920s and the 1940s.

Understanding the factors that influence mortality, and hence longevity, is an essential prerequisite for managing the risks associated with life-contingent exposures, such as pension liabilities, annuities and life insurance.

1.5 How this document is organized

This document is divided into three parts of nine chapters in total.

Part I provides the reader with an overview of LifeMetrics and an introduction to key demographic concepts and data sources associated with longevity and mortality risk management. The experienced reader may wish to skip most of Part I and move quickly on to Part II. However, we would recommend that at least the overview of LifeMetrics in Chapter 2 be read by all readers.

Part II gives a description of the LifeMetrics Index, together with a review of its current and historical performance. The discussion of the index includes the specification of the methods and algorithms used in its construction. More technical details and background is provided in Appendix A.

Part III is devoted to the LifeMetrics Framework. Chapter 7 provides a review of modeling methods for forecasting future mortality. Then Chapter 8 addresses the impact of mortality and longevity risk on different types of exposures, including pension plans, annuities and mortality-linked investments. Finally, Chapter 9 covers the subject of hedging and risk mitigation.
2. Overview of LifeMetrics

2.1 What is LifeMetrics?

LifeMetrics is a toolkit that provides a transparent framework for measuring and managing longevity and mortality risk. It can be used by pension plans, their sponsors, insurers, reinsurers and investors. It can be applied to a variety of portfolios and instruments for which mortality is a risk factor, including defined benefit pension liabilities, life-contingent insurance products, reverse (or equity-release) mortgages and life settlements. Furthermore, the objectivity and transparency of LifeMetrics provides a basis for capital markets-based solutions for hedging and transferring longevity and mortality risks that are currently being developed by JPMorgan and others.

The LifeMetrics toolkit consists of three components:

- **LifeMetrics Index**: Data for evaluating current and historical levels of mortality and longevity
- **LifeMetrics Framework**: A set of tools, methods and algorithms for measuring and managing longevity and mortality risk
- **LifeMetrics Software**: Software for forecasting future mortality rates

Together these components enable longevity and mortality risk to be measured in a standardized manner, aggregated across different risk sources and even reported as part of an enterprise risk management (ERM) program. For example, sponsors of defined benefit pension plans can quantify the potential impact of longevity risk on the solvency of their pension plans and the impact this could have on their overall organizational risk profile, cash flow and enterprise value. This toolkit also provides a means to evaluate the effectiveness of longevity/mortality hedging and risk transfer strategies, and to then quantify the residual risk. Furthermore it can be used by insurers to measure the extent to which the mortality risk in their life business offsets the longevity risk in their annuity business.

In summary, LifeMetrics can provide important input for evaluating decisions which involve the acquisition, transfer, hedging or retention of longevity and mortality risk.

2.2 LifeMetrics Index

2.2.1 Index summary

The LifeMetrics Index provides a transparent and objective window on current and historical longevity and mortality. It consists of various forms of mortality data for different countries that can be used as the basis for valuation of longevity-linked and mortality-linked exposures, forecasting future longevity and mortality rates, evaluating the risk associated with these exposures and determining the payoff of longevity derivatives and bonds.

The availability of a robust, consistent and transparent index is essential for effective management of longevity and mortality risk, and an important prerequisite for the development of a liquid longevity market.

The LifeMetrics Index is based on publicly available mortality data for national populations, broken down by country, age and gender. Initially, the LifeMetrics Index includes data from the US and from England & Wales. However, we intend to increase coverage and other countries will be available in the index in the months following its launch.

The LifeMetrics Index was designed to conform to certain characteristics widely acknowledged as essential for a successful index and critical for the development of a liquid market based on that index:

- **Objectivity**
- **Transparency**
- **Building-block approach to sub-indices**

Objectivity is considered a highly desirable characteristic of any index. The methods and algorithms used in the construction of the LifeMetrics Index were chosen to be as objective as possible, but without sacrificing the integrity of the underlying index data. Furthermore, the calculation of the index is not performed by JPMorgan, but by a calculation agent with expertise in the field of longevity and mortality analysis. Finally, the governance and oversight of the index is performed by an Advisory Committee comprised of individuals from different backgrounds and organizations in order to safeguard its integrity and objectivity.
In terms of transparency, the data sources, methodologies, algorithms and calculations used in the development and production of the index are fully disclosed and explained in this and other documents. These explanations include discussions of why the particular techniques and approaches employed were chosen ahead of various alternatives. In all cases these choices were made on the basis of a trade-off between simplicity, objectivity and transparency.

A building-block approach is as important to the LifeMetrics Index as it is to any index. Breaking the index down into a number of building blocks, or sub-indices, significantly increases the degree of applicability to a wide variety of situations and exposures. The building blocks that are used in the LifeMetrics Index have initially been chosen to include mortality data broken down by country, gender and age.

### 2.2.2 Index data

The data required to evaluate mortality and longevity involve two types of current and historical information collected by country, gender and age:

- The size of the population in question (the exposure to risk)
- The number of deaths occurring within that population in a given period

For the LifeMetrics Index these data are obtained from public sources, typically the governmental agencies mandated to collect and publish population and mortality data. Data from private sources, e.g., individual pension plans and insurers, are not included in the LifeMetrics Index at this time. These data are generally proprietary and not widely available to market participants.

The data published as part of the index include three different metrics:

- Crude central rate of mortality, $m$
- Graduated initial rate of mortality, $q$
- Period life expectancy, $e$

Each of these metrics is calculated and published for each year, as and when the underlying data become available. In addition, historical data are available for each of these metrics as part of the Index. The methods and algorithms used to produce these metrics are described in Chapter 5 and Appendix A.

The current and historical data published as part of the LifeMetrics Index give users a snapshot of the current state of mortality and longevity, together with valuable insight into the trends and volatility of historical changes in these variables.

### 2.3 LifeMetrics Framework

#### 2.3.1 Framework introduction

The purpose of the LifeMetrics Framework is to provide practical guidance on the implementation of programs to measure and manage longevity and mortality risks. It is intended to help users evaluate and apply mortality data to develop a risk management process appropriate to their organizational context. It should be noted that the framework is not limited in its application to data coming from the LifeMetrics Index. On the contrary, the framework is also broadly applicable to mortality data coming from other sources, such as individual pension plans, insurers and industry bodies.

The LifeMetrics Framework consists of a set of tools and methodologies for measuring longevity risk and mortality risk in a way that can be applied consistently to a wide variety of different situations, products and businesses. These include, but are not restricted to, the following:

- Defined benefit pensions, including retiree pensions and deferred pensions
- Life insurance, including endowments, term life and whole life
- Annuities, including immediate annuities, deferred annuities and variable annuities
- Life settlements
- Reverse, or equity-release, mortgages

Moreover, because mortality risk is measured in a consistent, standardized way, regardless of its source, the LifeMetrics Framework enables users to calculate the aggregate mortality risk across different products, businesses
and populations. This makes the management of mortality risk a much more viable and more strategic enterprise than it has been in the past.

Applying the LifeMetrics Framework equips organizations with the ability to evaluate and optimize mortality risk management decisions which involve:

- Hedging
- Risk transfer
- Increasing risk exposure

As a result, informed decisions can be made from an enterprise-wide perspective, taking a holistic view of mortality risk and its impact on the overall risk profile of the organization. In particular, for organizations with defined benefit pension plans, the LifeMetrics Framework helps sponsors, fiduciaries and trustees address issues such as:

- Continuance or closure of the pension plan for active members
- Buyout (i.e., externalization) of the plan
- Risk management
- Funding
- Impact of plan decisions on shareholder value
- Impact of plan decisions on beneficiaries

Furthermore, for insurers and reinsurers the framework can complement their existing tools and provide additional input for decision-making in areas such as:

- New business development
- Growth strategies for existing businesses
- Risk management
- Capital allocation

Finally, the framework assists investors in evaluating the trade-off between expected return and risk for longevity-linked and mortality-linked investments, as well as the diversification benefits relative to other asset classes. This enables them to evaluate, on a portfolio basis as well as a stand-alone basis, investment decisions related to:

- Buy strategies - increasing exposure
- Hold strategies - maintaining exposure
- Sell strategies - reducing exposure

Moreover, it provides the basis for investors to move towards more focused investment strategies based on the longevity profile of particular age groups, socio-economic groups and other subpopulations where they may identify promising investment opportunities.

Overall the development of new capital markets solutions for transferring longevity/mortality risk - that we are actively promoting - will ultimately enable all these decisions to be implemented economically and efficiently.

2.3.2 Risk management framework

The LifeMetrics Framework can be summarized as a five-step approach to risk management, illustrated in Figure 2.1.

The first step involves understanding the nature of the underlying exposure to longevity/mortality risk. This means analyzing how longevity and mortality impact the value of the exposure. It includes understanding the nature of the underlying population, in particular size and demographics, as well as evaluating specifically how the exposure depends on mortality rates. These issues are addressed in more detail in Chapter 8.

The second step involves sourcing the appropriate data on the longevity and mortality profiles of the population specific to the exposure, as well as additional data required to determine the value and risks associated with that exposure (e.g., interest rate data, inflation data, etc.). In this step it is important to understand the shortcomings of the data that is being used, particularly in terms of:

- Accuracy
- Timeliness
- Available history
- Methodologies for collection and calculation
- Major sources of error
- Treatment of missing and incomplete data.

It is often the case with mortality data that compromises need to be made. For example, a pension plan’s own historical longevity experience is clearly the data most appropriate for its particular population of beneficiaries; however, there may only be a very short history available, and/or the quality and reliability of that data might be poor. By
contrast, national population data do not accurately match the characteristics of the population of beneficiaries, but do provide a longer history and greater accuracy. The pension plan then faces a trade off between using its own more relevant but lower quality data versus using higher quality data relating to a different population. In practice it is usually appropriate to use a combination of data sets, suitably adjusted for their deficiencies, to come up with a relevant longevity analysis. Understanding the so-called “basis risk” between different populations is an important part of this process and is a key element in the LifeMetrics Framework. Population basis risk is discussed in Chapter 9.

Two further challenges in dealing with mortality data involve:

- Addressing the noise inherent across the “mortality curve” at a point in time
- Estimating mortality rates at high ages

The first of these issues typically leads to a lumpy mortality curve when crude mortality rates are plotted against age at a given time. Furthermore, the lumpiness does not persist from year to year but changes randomly due to noise in the data collection process and the mortality process itself. Techniques for dealing with this issue by smoothing or “graduating” the mortality curve are discussed in Appendix A. Then in Chapter 5, we define the graduation method currently implemented in the LifeMetrics Index.

The second challenge described above refers to the difficulties in measuring mortality accurately at high ages. This is problematic because the relatively small number of people alive at very high ages means that measured mortality rates have large statistical errors. For example, if for a particular year there is only one person alive aged 110, then the observed mortality rate for that age in that year can only be either 0% or 100%. In other words, the observed rate does not reflect the true underlying probability of dying. In Appendix A we discuss the various methods used to deal with this problem and in Chapter 5 we define the method that has been currently implemented in the LifeMetrics Index.

The methods described in Appendix A will also enable users to transform raw, or “crude”, mortality data from sources other than LifeMetrics into suitable measures of longevity and mortality that are appropriate for pricing, modeling and risk management.

The third step in the LifeMetrics Framework involves forecasting future longevity and mortality so that exposures can be valued and their risks calculated. At its simplest, users can develop a single expected, or “best estimate”, path for future mortality rates, taking account of expected mortality improvements, together with a “worst case” path that has an adverse impact on the exposure.

At the other extreme, sophisticated users will develop stochastic models for forecasting future longevity and mortality. This means projecting mortality rates into the future using so-called stochastic projection models, which are discussed in greater detail in Chapter 7. Based on historical data and judgements about the future trend of mortality rates, these projection models simulate a large number of possible paths (or scenarios) for mortality rates around a central, expected path. Collectively the set of paths facilitate the calculation of uncertainties in future mortality rates. In addition to these uncertainties, one should also use stress tests to shock the expected path and evaluate the robustness of the forecast to “worst case” outcomes. This is explained more fully in Chapters 7 and 8, as well as in a separate working paper (Cairns et al., 2007).

The fourth step in the LifeMetrics Framework involves the quantification of the impact of longevity and mortality risk on the particular exposure being considered. In particular, for a defined benefit pension plan, this step means evaluating the sensitivity of the value of pension liabilities to changes in longevity, where the changes are plausibly consistent with historical data and/or expectations. The set of paths for future mortality calculated in the previous step lead to a set of paths for the future value of pension liabilities and the uncertainties associated with them. Having thus quantified the amount of longevity risk in the pension in monetary terms, this becomes an important input for deciding what action should be taken, if any, to alleviate it. This risk quantification step is discussed in more detail in Chapter 8.

The fifth step in the LifeMetrics Framework involves evaluating alternative courses of action in dealing with the longevity and mortality risk associated with the exposure.
This is essentially a trade-off between the risk and reward of the different strategies. For example, the decision to invest or increase exposure to longevity should be evaluated by weighing up expected return against the incremental risk associated with this particular exposure, given the specific demographics and price. Similarly, the decision to hedge longevity or mortality risk also needs to be addressed in these terms. Hedging removes some or all of the risk, but carries a cost. Doing nothing avoids the cost of hedging but leaves a potentially large risk exposure that could have devastating consequences. As part of this step, hedgers should evaluate the residual risks that remain after the hedge is put into place. These residual risks may include population basis risk and hedge roll risk. The issues connected with hedging longevity and mortality risk include hedging longevity and mortality risk - including hedge effectiveness and minimizing residual risk - are presented in Chapter 9.

2.3.3 Framework tools, methods and algorithms

The chapters that follow describe the various elements in the LifeMetrics Framework. This includes, in particular, the tools, methods and algorithms alluded to in the previous section, which are essential in enabling users to:

- Transform raw or crude mortality data from any source into appropriate mortality rates, probabilities and other metrics suitable for valuation and risk management
- Understand the actuarial and demographic literature on longevity and mortality modeling
- Develop and calibrate forecasts of future mortality rates
- Develop and calibrate stochastic simulation models for mortality rates
- Evaluate the sensitivity of exposures (e.g., pension liabilities, annuity portfolios, etc.) to longevity and mortality risk and quantify the risk in monetary terms
- Compare alternative strategies in terms of expected reward and risk
- Minimize population basis risk and other residual risks associated with any hedging strategy
- Evaluate hedge effectiveness

2.4 LifeMetrics Software

The third element of LifeMetrics is software. We are making software available to assist with the most complex aspects of longevity/mortality risk management, namely, building, calibrating and implementing models for analysis and fore-

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Figure 2.1 The LifeMetrics Framework

1. Determine nature of exposure
   - Analyze population size and demographics
   - Evaluate dependence of exposure on mortality rates
2. Source data
   - Compile appropriate data
   - Understand data shortcomings
   - Analyze basis risk
   - Graduate data
3. Forecast future longevity / mortality
   - Develop forecast
   - Simulate possible paths for future mortality
   - Perform stress tests
4. Quantify risk
   - Evaluate sensitivity to mortality rates
   - Price the exposure
   - Quantify risk in monetary terms
5. Decide course of action
   - Compare different courses of action in terms of risk and reward
   - Evaluate residual risk
casting. This software can be downloaded from the LifeMetrics website.

The software comprises of a number of stochastic mortality models, namely:

1. Lee-Carter model (1992)
5. Extensions to the Cairns, Blake and Dowd model (Cairns et al., 2007)

A brief discussion of these models, along with a review of alternative approaches, is contained in Chapter 7. In addition, a full technical description of the models and a quantitative comparison of their dynamics is contained in a working paper (Cairns et al., 2007), which is also available for download from the LifeMetrics website.

2.5 Evolution of LifeMetrics

LifeMetrics is not intended to be a rigid or prescriptive platform for managing longevity and mortality risk. Rather it is an inclusive platform that embraces a diversity of tools, which are applicable to different situations. As such, it will evolve and expand as new models, new methods and new algorithms come to light. As new tools are developed and proven by us, our advisors and other parties, they will be incorporated into the LifeMetrics toolkit.

The definition and construction of the LifeMetrics Index described herein has been based on the requirements of transparency, objectivity, consistency and accuracy, given the tools available and the known limitations of the data. Over time the methodology for producing the index will need to adapt as new countries are added and new situations and new data sources arise. This will be overseen by the LifeMetrics Index Advisory Committee who will safeguard the integrity of the index.
3. Key concepts in longevity and mortality data

3.1 Introduction

Mortality data are a form of demographic data reflecting the death statistics associated with a specified population. The population may be as broad as the national population of a country if we are considering national mortality, or much narrower, such as the life insurance policyholders of an insurer, the annuitants of an annuity provider, or the members of a defined benefit pension plan. In all cases, mortality data are largely meaningless without an understanding of the population to which they refer, in terms of its socio-economic and lifestyle characteristics.

As we have already mentioned, the data required to evaluate mortality essentially involve two types of demographic information:

- The total size of the population in question at different points in time
- The number of deaths occurring within that population within different periods of time

For a life insurer, annuity provider or pension fund, data on the population of policyholders, annuitants or members can be very detailed and of a high quality, making mortality studies of insured lives, annuitants or pension plan members straightforward and accurate. However, for a country as a whole this is generally not the case.

Because reporting deaths is a legal requirement, national death data are generally recorded and documented accurately in developed countries. However, national population data are problematic. Most countries only conduct a formal census every ten years or so, and therefore only get an accurate snapshot of the population at that time. Between censuses the size of the population needs to be estimated based on a variety of demographic data including:

- Most recent census data
- Birth data
- Death data
- Migration data (immigration and emigration)

Mortality data on the population as a whole is of limited use unless it can be broken down into subpopulations, in which individuals are grouped according to common characteristics. The commonalities among members of such subpopulations provide greater insight into the drivers of mortality and greater confidence in forecasting longevity. The population characteristics that are relevant to mortality and longevity include the following:

- Age
- Gender
- Marital status
- Social class
- Lifestyle (e.g., smoker vs. non-smoker)
- Employment
- Health
- Regional geography (e.g., postcode or zip-code)

Clearly mortality data that can be broken down by all of these characteristics simultaneously would be ideal from the point of view of understanding population demographics and mortality. While insurance companies and pension plans have access to many of these characteristics for their particular policyholder and member populations, such detail is generally not available for national populations. This is for two reasons. First, it may not be collected at the appropriate level of detail by the relevant government agency. Second, even if it is collected, it may not be publicly published at this level of detail, in order to protect the privacy of individuals who may be easily identified within the very small subpopulations corresponding to a particular age, gender, social class and geography. However, mortality data for national populations are available for a large number of countries broken down by age and gender.

3.2 Some demographic concepts

There are a number of concepts that are useful in understanding the data associated with mortality and following the literature on the subject. In this section we describe a few of the most important.

3.2.1 Birth cohort

A “birth cohort” — or simply “cohort” — is a group of individuals who were born in the same period of time, generally
the same calendar year. So demographic data by cohort tracks the subpopulation of individual lives born in the same calendar year.

An example of the importance of cohorts as a concept is provided by the so-called “cohort effect” that exists in mortality data for England & Wales. The “cohort effect” refers to the observation that people in the population of England and Wales who were born in the 1930s exhibit higher rates of mortality improvement than those born in both the 1920s and the 1940s. The longevity of the 1930s cohorts is an important factor in forecasting mortality improvements for England & Wales and is widely discussed in the literature (see, for example, Richards, 2007).

3.2.2 Age last birthday

“Age last birthday” is a self-descriptive concept that treats the age of an individual as a whole number of years (i.e., 0, 1, 2, 3, …). It essentially rounds the exact age of the individual down to his or her age on the most recent birthday. For example, the age last birthday of someone born on 1 January 1931 will be 75 from 1 January 2006 until 31 December 2006 inclusive.

When dealing with a population, then the “sub-population aged 75” at a particular point in time refers to the group of individuals who were aged 75 at the last birthday. So it includes people whose exact ages fall between 75 and 75 + \( \frac{364}{365} \). More generally, the “sub-population aged \( x \)” at a given point in time will include individuals whose exact ages range from \( x \) years to \( x + \frac{364}{365} \) years. Note that in studying mortality, age is generally denoted by \( x \).

3.2.3 Exposure-to-Risk

Another important concept is what is called “exposure-to-risk”. The concept of “exposure-to-risk” of a population over a period of time (e.g., a calendar year) is very similar to the average number of individuals in that population over that period.

More precisely, the exposure-to-risk in a given year refers to the total number of person-years in that population during that calendar year. This is an important concept as individuals can join or leave the population over the course of the year. For the national population, for example, individuals join or leave due to births, deaths, immigration and emigration. For a life insurer, individuals join the insured population when they buy a policy (this is effectively equivalent to immigration) and they leave the population when the policy lapses, is cashed in or matures (all effectively emigration), or when the individual eventually dies. Similarly for a pension plan, individuals join the population when they join the plan and leave it when they transfer their benefits (emigration) or die.

As an example, an individual who emigrates (leaves the population) on 1st July contributes only 0.5 person-years to the exposure-to-risk for the population for that year. Similarly, an individual who dies on 1st April contributes only 0.25 person-years to the exposure-to-risk.

The exposure-to-risk concept is illustrated in Figure 3.1. The lines in the diagram represent individual lives in the population over the calendar year in question, which in this case is 2006. They are called life-lines. Each life-line either enters 2006 from the previous year, or originates within 2006 as the corresponding individual joins the population (due to birth or immigration). Similarly, each life-line either leaves 2006 at year end, or terminates within 2006 as the individual leaves the population (due to death or emigration). In this example, the initial population at the start of the year is 5 individuals, the population at year-end is 4, and the exposure-to-risk is 4.33. Note that three different
metrics for characterising the average population over the period — the mid-year population (5.00), the simple average between start and end of year population (4.50) and the exposure-to-risk (4.33) — are reasonably consistent with one another but differ slightly.

The concept of exposure-to-risk also applies in an obvious way to sub-populations. For example, the “exposure-to-risk at age 75 in 2006” refers to the total person-years lived in the age range 75 to 75 + 364/365 in the calendar year 2006. Note that in addition to the reasons listed above (birth, death, immigration and emigration), individuals will also join and leave this sub-population as they pass through a birth-day.

3.2.4 Lexis diagram

A Lexis diagram is a two-dimensional plot of a population’s life experience by time and age. See the example in Figure 3.2. This diagram is similar to the diagram in Figure 3.1, which was discussed in the previous section, but it includes the age dimension as well as the time dimension. One consequence of including both age and time is that the life-lines of individual lives are now diagonal, running from the bottom left to the top right.

The Lexis diagram is named after the German actuary and statistician Wilhelm Lexis (1837-1914) and plays a valuable role in demographics by virtue of the simple and intuitive visual representation it provides of the information relating to groups of individuals and the events that are relevant from a population and mortality perspective.

As can be seen in Figure 3.2, the Lexis diagram is segmented into cells 1-year in time by 1-year in age. The 45 degree diagonal lines represent the lives of individuals which begin at birth or with immigration into the population (denoted by an open circle) and terminate at death (denoted by a cross) or emigration out of the population (denoted by a solid circle). Each time-age cell contains individual lives with the same age last birthday over the calendar year. In particular, the white cell in Figure 3.2 contains individuals who in the year t were aged x on their last birthday.

In the ideal situation where the life-lines of all individuals are known exactly, the Lexis diagram gives a very clear graphical representation of the size of the population and the exposure-to-risk for each time-age cell. In particular, the year-start and year-end measures of the population size are simply given by the number of life-lines that cross the cell’s left-hand and right-hand boundaries respectively. By contrast, the exposure-to-risk for a cell is just the sum of the lengths of the portion of each life-line that falls within that cell (divided by the square-root of 2 since life-lines are 45 degree diagonals). The mortality rate for the cell, therefore, is just the number of deaths (crosses) falling within the cell divided by the exposure-to-risk.

Unfortunately in many real-world situations in which mortality is being measured in order to manage the risks associated with national populations, life-lines are not known exactly. In this case the exposure-to-risk can only be estimated from the population size at particular times. For example, if population estimates are known at the start and end of the year, then one simple estimate of exposure-to-risk is just the arithmetic average of the two.

3.3 Measures of mortality and longevity

3.3.1 The initial rate of mortality q and survival p

A measure of mortality typically quoted in a standard mortality table is the probability that a person aged x dies within the next year. This probability is referred to as the “initial rate of mortality” and is generally denoted by q_x. In the absence of data constraints, the obvious way to calculate q_x is to start with a closed population of lives aged x, count the number of deaths over the year, then divide the number of deaths by the number of lives at the start of the year.
In practice the calculation is not as straightforward as this, because census and death databases record numbers for an open population, that is, one that is also impacted by migration.

Another important measure of mortality that is closely linked to \( q_x \) is survival probability \( p_x \), which reflects the probability that a person aged \( x \) survives over the next year. Clearly, survival probability is given by 1 - probability of dying, that is:

\[
p_x = 1 - q_x
\]

### 3.3.2 The central observed rate of mortality, \( m \)

In practice the data generally used for calculating population mortality are:

- The number of deaths over the year of individuals who were aged \( x \) last birthday
- The number of lives aged \( x \) last birthday at the middle of the year, which serves as a proxy for the exposure-to-risk

Dividing the first of these by the second produces an estimate for the “central rate of mortality”, denoted \( m_x \), from which \( q_x \) can be approximated.

The central rate of mortality reflects deaths per unit of exposure over an entire year. Using lives at the middle of the year as an approximation to the exposure to mortality risk over the year assumes that the population changes uniformly over the year.

This measure of mortality is different from the probability measure because the denominator for \( m_x \) only counts exposure to death for the fraction of the year for which the person was alive. By contrast the \( q_x \) denominator counts exposure to death as a whole year whether or not the person survives the entire year.

It follows that a simple approximation for \( q_x \), assuming a uniform distribution of deaths over the year, is

\[
q_x = \frac{m_x}{1 + \frac{1}{2}m_x}
\]

### 3.3.3 The force of mortality, \( \mu \)

Another measure of mortality often quoted is \( \mu_x \), the so-called “force of mortality”. This is the instantaneous death rate for lives aged \( x \) exactly. So the distinction is as follows: whereas \( q_x \) is the death rate over one year, \( \mu_x \) is the instantaneous death rate. To illustrate the relationship between these two mortality measures define \( \mu_{x+t} \) to be the instantaneous death rate for lives aged \( x+t \) exactly at time \( t \), where \( t \) can be a fractional year. Similarly define \( l_{x+t} \) to be the number of exposed lives aged \( x+t \) exactly. Then \( \mu_x \) and \( q_x \) are related in terms of the expected number of deaths over the next year of people who were aged exactly \( x \) at the start of the year

\[
\int_0^1 l_{x+t} \cdot \mu_{x+t} dt = l_x \cdot q_x
\]

where the integral is over the next year. Simple approximations to \( \mu_x \), assuming a uniform distribution of deaths over the year, include the following:

\[
\mu_x = -\log(1 - q_{x+\frac{1}{2}})
\]

and

\[
\mu_x = m_{x+\frac{1}{2}}
\]

### 3.3.4 Eliminating noise from mortality rates

The mortality rates described above are sometimes referred to as “crude rates” because they are based on raw unadjusted mortality data. In different practical applications, however, mortality rates are often averaged over time and/or smoothed across ages.

In order to improve the statistical accuracy of crude mortality rates, it is common for published rates for certain purposes to be calculated using data over, say, a two-year period rather than just a single year, e.g., rates for 2001 calculated from 2000-2001 data inclusive. This averaging over time smoothes the year-to-year noise in crude mortality rates and diminishes the impact of spurious single-year peaks and troughs.

Furthermore, most standard mortality tables quote mortality rates that have been smoothed, or “graduated”, across
different ages. The crude mortality rate at age \( x \) would be a suitable estimate of the underlying “true” mortality rate at a particular time if there were no information about mortality rates at adjacent ages. However, the availability of crude rates across the whole spectrum of ages suggests that better estimates of the “true” mortality rate at age \( x \) can be obtained by taking crude rates at adjacent ages into account. The method of doing this is effectively a smoothing process and, in this context, is called graduation. Numerous different techniques for graduation have been devised over the years, including methods involving weighted averages across ages, cubic splines and more complex smoothing approaches (see Appendix A).

Graduation of mortality rates is useful because the raw mortality data can contain isolated features that cannot be explained rationally, as well as other sources of statistical noise. In particular, it is generally accepted that beyond the younger ages, mortality rates increase with age. However mortality rates calculated from raw data could, for example, suggest counter-intuitively that 50 year olds have a higher mortality rate than 51 year olds. A graduated mortality table would normally smooth through these local peaks and troughs to ensure that such inconsistencies are not present in published mortality rates.

The graduation of mortality rates is discussed in greater detail in general terms in Appendix A, and its application to the LifeMetrics Index is explained in Chapter 5.

### 3.3.5 Life expectancy, \( e \)

Life expectancy is a mortality measure that equates to the average future lifetime of an individual at a given age. The term is used interchangeably with “life expectation” or “expectation of life”.

Algebraically life expectancy at age \( x \) can be written as:

\[
e_x = \sum_{t=0}^{\infty} t \cdot p_x
\]

where we assume 120 is the ultimate age and \( p_x \) is the relevant survival rate defined as the probability that a person aged \( x \) survives \( t \) more years:

\[
i \cdot p_x = \prod_{i=0}^{t-1} (1 - q_{x+i})
\]

Note that the life expectancy formulae shown here relate to the calculation of “curtate” period life expectancy, where future lifetime is a discrete random variable as opposed to a continuous one. In other words, in calculating the expected value of future lifetimes, only whole numbers of future years are assumed. In the remainder of this document, life expectancy will always be used to mean curtate life expectancy. As should be evident from the above formula, different survival rates, \( p \), (or equivalently different mortality rates, \( q \)) produce different levels of life expectancy. There are two general types of life expectancy statistics which are based on current and projected mortality rates, respectively. They are:

- Period life expectancy
- Cohort life expectancy

Published life expectancy statistics often relate to mortality rates from what is known as a “period” life table (see next section). More precisely, such a statistic is known as “period life expectancy”. This means that the mortality rates used to calculate life expectancy are those corresponding to the current period and do not explicitly take account of how mortality rates are expected change in the future. For example, in these calculations, the age 90 mortality rate that is assumed in 70 years’ time for today’s 20 year olds is the same rate as that for a 90 year old today. Period life expectancy can be useful as a headline indicator because it is an objective measure and avoids the subjectivity inherent in forecasting future mortality improvements.

For estimating the average future lifetime of an individual however, it is more relevant, albeit more subjective, to calculate life expectancy allowing for expected future changes in mortality. This requires a “generational” or “cohort” life table (see next section). With a cohort life table, a separate set of mortality rates can be projected for each year for each birth cohort. Hence the age 90 mortality rate applying to today’s 90 year olds will differ from that applying in 70 years to today’s 20 year olds. This form of life expectancy is known as “cohort life expectancy” and leads to results that are typically higher than period life expectancy. The calculation is similar to period life expectancy, but using the following formula:
Regardless of the underlying mortality rates of a life expectancy, there is a relationship between life expectancy and average age at death which is worth noting, namely, average age at death = \( x + e_x \). Holding all else constant, as age \( x \) increases, life expectancy typically decreases while average age at death increases.

3.4 Life tables (Mortality tables)

3.4.1 Description

A life (mortality) table is a compilation of information related to the probabilities of mortality or survival for a given population. One main application of a life table is in calculating the value of liabilities that are contingent upon mortality. This document identifies various forms of mortality tables. Described first is the most basic table, called a period life table. Extensions of the period life table are then described in the sections that follow: select and ultimate tables, generational tables, and multiple decrement tables.

3.4.2 Period life tables

The simplest and most common form of life table is the period life table. In its most basic form, a period table is a listing of ages and, for each age, the corresponding probability of death within the next year. It is called a “period” life table because it is a snapshot of mortality rates over a specific period, the period typically being a calendar year.

\[ i\hat{p}_x = \prod_{j=0}^{j-1} (1 - q_{x+j,i}) \]

It does not make assumptions about how mortality rates might change in the future.

The standard notation for the initial mortality rate is \( q_x \), where \( x \) is age. For example, given the life table shown in Table 3.1, the probability of an 89 year old dying in the next year is 0.1 (i.e., 10.000%).

Although this table provides the initial mortality rates only classified by age, tables are often produced which distinguish between other demographic factors, like gender. Separate mortality rates are normally produced for males and females because females generally have lower mortality rates than males.

In addition to showing mortality rates, many life tables also present other variables related to mortality rates. The simplest variable is the probability of survival over the next year. For an individual aged \( x \), the notation is \( p_x \). Additional variables may also be presented in period life tables, although they are less commonly used and will not be discussed here.

Period life tables for standard use are usually calculated from raw historic demographic data and then smoothed or “graduated” to ensure an intuitive progression of mortality rates by age. Because period life tables are built from actual mortality experience, they relate to a particular period in history and so are superseded by new tables as more up-to-date data becomes available.

<table>
<thead>
<tr>
<th>Age (x)</th>
<th>Initial mortality rate, ( q_x ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.0500</td>
</tr>
<tr>
<td>21</td>
<td>0.0520</td>
</tr>
<tr>
<td>22</td>
<td>0.0540</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>89</td>
<td>10.0000</td>
</tr>
<tr>
<td>90</td>
<td>11.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Age [x]</th>
<th>( q_{00} ) (%)</th>
<th>( q_{00+1} ) (%)</th>
<th>( q_{00+2} ) (%)</th>
<th>Age [x+2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Select year 1</td>
<td>0.0300</td>
<td>0.0420</td>
<td>0.0540</td>
<td>22</td>
</tr>
<tr>
<td>Select year 2</td>
<td>0.0320</td>
<td>0.0440</td>
<td>0.0560</td>
<td>23</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>89</td>
<td>8.5000</td>
<td>10.5000</td>
<td>12.0000</td>
<td>91</td>
</tr>
<tr>
<td>90</td>
<td>9.5000</td>
<td>11.5000</td>
<td>13.0000</td>
<td>92</td>
</tr>
</tbody>
</table>
There are currently numerous period life tables available in the public domain which relate to a wide variety of populations. Within the UK, for example, there are general population tables for England & Wales in 10-year intervals going back to 1841. There are also tables going back to the 1920s based on the population of insurance policyholders.

### 3.4.2.1 Select and ultimate tables

Different users of life tables have found that not all of the pertinent information on mortality can be captured in a period life table. An extension of the period life table is the select and ultimate table, which is often used by insurance companies.

For insurance purposes, some life tables have been published with one or more “select” rates and an “ultimate” rate for each age. When a person initially takes out an insurance policy, he will normally have been underwritten for good health. In effect, the insurance company will “select” healthier lives from a larger population. As time passes, the insurer’s knowledge of the policyholder’s health declines. As a result, the mortality rate for these “select” lives in the early years of the policy generally will be lower than for policyholders of the same age who had been underwritten a long time ago. Those who were underwritten a long time ago are eventually deemed to experience their “ultimate” mortality rates as there is no distinguishable difference in mortality between someone underwritten 9 years ago from someone underwritten 10 years ago.

For practical reasons, insurance life tables with a select and ultimate feature might only have one or two select rates. Such a table, if it had two years of select rates, would then take the form shown in Table 3.2.

The way to read Table 3.2 is as follows. For a life aged 20 who has just taken out a policy, the mortality rate over the first year is 0.030%. In the second year, when the life is then aged 21, the rate is 0.042%. In the third year, when the life is then aged 22, the rate is 0.054%. In the fourth year when the life is then aged 23, the rate is 0.056%.

### 3.4.2.2 Generational (Cohort) life tables

One aspect that both the period life table and the select and ultimate table ignore is how mortality rates may change in the future. Generational tables, described here, explain how future changes in mortality may be taken into account.

Whereas select and ultimate tables have different mortality rates depending on the policy year (years since the insurance policy was issued), generational tables have different mortality rates depending on the calendar year. So a 20 year old in 2007 will have a different mortality rate to a 20 year old in 2008. Unlike select and ultimate tables where the select period only lasts for a few years before reaching ultimate, generational tables normally have mortality rates changing indefinitely with time.

Generational tables are useful because it is generally accepted that mortality rates are improving over time at most ages and not to take this into account could materially underestimate the value of an annuity policy or pension liability. For a life insurer, not taking into account future mortality improvements could materially overestimate the value of a life insurance policy. An example of a generational table organised by calendar year is shown in Table 3.3.

In the notation for generational tables, \( q_{x,yyyy} \) represents the mortality rate of an individual aged \( x \) in calendar year \( yyyy \). Generational tables are sometimes called cohort tables because they can follow a cohort of lives over time, where each cohort is defined by its year of birth. In order to follow the cohort aged 20 in 2006 (or born in 1986), the table above would need to be read diagonally from top left to bottom right. The rate for the 20 year old in 2006 is 0.050%. In 2007, the person will be aged 21 and the rate will then be 0.051%, and so on.

Generational tables can also be organised by cohort, so instead of calendar year as the column heading, each col-

<table>
<thead>
<tr>
<th>Age (x)</th>
<th>( q_{x,2006} ) (%)</th>
<th>( q_{x,2007} ) (%)</th>
<th>…</th>
<th>( q_{x,2099} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year: 2006</td>
<td>Year: 2007</td>
<td>…</td>
<td>Year: 2009</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.0500</td>
<td>0.0490</td>
<td>…</td>
<td>0.0200</td>
</tr>
<tr>
<td>21</td>
<td>0.0520</td>
<td>0.0510</td>
<td>…</td>
<td>0.0210</td>
</tr>
<tr>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
</tr>
<tr>
<td>89</td>
<td>10.0000</td>
<td>9.9900</td>
<td>…</td>
<td>8.5000</td>
</tr>
<tr>
<td>90</td>
<td>11.0000</td>
<td>10.9000</td>
<td>…</td>
<td>8.0000</td>
</tr>
</tbody>
</table>
umn heading could be defined in terms of year of birth. In this case, the table would be read downwards and not diagonally in order to follow a particular cohort over time. The notation for a generational table organized by cohort would be changed to $q_{x,zzzz}$, representing the mortality rate of an individual aged $x$ who was born in $zzzz$.

Another way of presenting generational tables is to start with a base table, which has the mortality rates applicable for the first year as though it were a period life table, and then provide a separate two-dimensional table of mortality improvement factors. Computationally, this can be more convenient if the mortality improvement factors can be represented by a formula that is a function of age and time. In this case, there is the potential to create a multitude of tables very simply by changing the parameters in the formula, but without having to store every single table that might be of interest.

This in fact was the approach adopted in the UK before a set of three standard generational tables were produced by the Continuous Mortality Investigation Bureau (CMIB), called Short Cohort, Medium Cohort and Long Cohort. These are based on three sets of assumptions about how long the trend of higher mortality improvement rates would continue before flattening off, Short Cohort having the smallest level of mortality improvements over the long term and long cohort the greatest.

A cautionary word about generational tables is that mortality improvement rates are notoriously difficult to predict and in publishing standard tables its users may be misled into assuming that the projections are sufficiently accurate. The UK insurance industry gravitated towards using Medium Cohort as a benchmark mortality table, whereas some research has suggested that these tables still under-estimate future mortality improvements.

3.4.2.3 Multiple decrement tables and multi-state tables

In most situations mortality is not the only form of decrement (way of leaving) for a population. Other forms of decrement may be surrender of an insurance policy, withdrawal from a pension plan, and so on. A population can also be separated according to factors or “states” that materially affect the value of the insurance policy of pension obligation, such as state of health or marital status.

Multiple decrement and multi-state tables are more complex tables that take account of more ways of leaving a cohort than death, or more states of existence than merely being alive or dead. For example a multi-state table might also indicate the probability of a person moving from the state of being alive to being ill. Multi-state tables here are analogous to rating transition matrices in the context of credit. These tables have their uses in certain situations but for mortality studies generally they involve more complexity and data issues than what is practical to address at this stage and will not be considered further in this document.
4. Sourcing longevity and mortality data

4.1 Sources of mortality data

In general there are three different potential sources of mortality data:

- Mortality data of individual pension plans, annuity providers and life insurers on their specific populations
- Mortality data on aggregate insured lives, annuitants and pension populations collected by industry bodies
- Mortality data collected by governments on the national population

Note that the populations associated with each of these potential data sources is a subset of the population associated with the data source listed below it. Furthermore, as a data source for studying mortality and longevity, each of these has a number of pros and cons in relation to the usefulness of the associated mortality data.

4.2 Mortality data for specific pension plans, life insurers and annuity providers

Most large pension plans, life insurers and annuity providers have detailed, good quality data on the mortality experience on their own particular populations. These data provide the best source of information for forecasting future mortality for these populations and for projecting future solvency. Clearly the detail, quality and tailored nature of the mortality data are significant advantages in understanding and forecasting mortality for these populations. However, there are also disadvantages associated with this data source. First, it is not public and therefore not available to others. Insurers in particular, jealously guard their experience data and see it as a critical source of competitive advantage. Second, for many small or medium-sized insurers and pension plans, mortality rates cannot be measured accurately because the size of the population is relatively small and, consequently, the statistical (sampling) errors in calculated mortality rates are high. This problem is exacerbated at high ages, at which fewer individuals are living. Third, the record keeping and data acquisition processes of many organizations are not robust or accurate and can lead to delays in recording death events or calculating exposure-to-risk, resulting in inaccuracies in calculated mortality rates.

4.3 Aggregate mortality data compiled by industry bodies

By contrast, aggregate mortality data collected from a number of different pension plans and insurers by industry bodies can overcome some of the above disadvantages. The UK’s CMIB (Continuous Mortality Investigation Bureau) is an example of such a body. It collects separate data on insured lives and annuitants from life insurers and data on pensioner lives from pension plans. By aggregating data in this way and then publishing it without reference to the companies taking part in the survey, there is no longer a confidentiality issue with making the data public. Furthermore, even insurers and pension plans with small populations can be dealt with effectively as they are combined into larger populations. However, the key disadvantage of this kind of data is that the organisations that contribute data (and therefore the underlying populations) change from year to year and survey to survey. This introduces undesirable noise in the data and makes it very difficult to compare mortality rates from year to year on a consistent basis. Furthermore these data are not published every year, which limits their value as an index.

4.4 National population mortality data

National population data overcome many of the deficiencies described above. In particular, these data are publicly available, comprise large numbers of individuals (and therefore have low sampling errors), have long histories and involve consistent, systematic procedures for collection, analysis and publication. As such, national population mortality data provide the most appropriate information for analysis of long-term trends and uncertainties attached to mortality.

National mortality data are published by government agencies or departments. For example, in the US data on national deaths are published by the Centers for Disease Control and Prevention (CDC) and data on the national population are published by the US Census Bureau. In the UK, it is the...
Office for National Statistics (ONS) that now publishes data on population, deaths and mortality rates. National mortality data are also republished for a large number of countries on the Human Mortality Database (HMD) website at www.mortality.org.

Despite the above advantages, national population data also have several disadvantages and shortcomings. First of all, it is generally less detailed than insurer-specific and pension plan-specific data in terms of being able to be broken down by different population characteristics (such as social class, lifestyle, geography, health, etc.). This is because such breakdowns are either not published to protect individual privacy or not collected in the first place. Second, data on the size of the population and the exposure-to-risk are not accurate away from census dates because of migration and this can lead to errors in the mortality rate. Third, despite the fact that deaths must be reported by law, there are inaccuracies in collecting death data. The age at death might be unknown and not recorded, or it might only be a guess on the part of the person reporting it. In the latter case there is a tendency for the ages of the elderly to be exaggerated, or rounded to the nearest five or ten years. Fourth, the mortality experience of the national population might not accurately reflect the mortality experience of the population being studied, leading to what is called a population basis risk.

4.5 England & Wales national mortality data

4.5.1 UK Mortality

4.5.1.1 Annual death data - collection

Death data for England & Wales are compiled by the Office for National Statistics (ONS) Mortality Statistics department. They have historical datasets of daily death numbers from 1970 to 2004 by individual age, sex and year of registration. From 1959 to 1969 death data are available monthly rather than daily.

Mortality statistics for England & Wales started in 1841. Prior to 1959 data are only available in five- or ten-year data buckets, although data in electronic format starts from 1901.

For more details see “Historic Mortality Tables - 1837 to latest year and Notes to Tables”, available on the ONS website.

Deaths are recorded for the country in which they occur. For example an English person dying abroad on holiday is not counted in the England & Wales data, but a foreigner dying in England while on holiday is counted.

For England & Wales death statistics are published by occurrences from 1993 and by registrations before this date. Occurrence data relate to deaths that occurred in a particular year even though some of the deaths included may be registered after the end of that year. Registration data relate to deaths that were registered in a particular year and will normally exclude some deaths that have occurred but not yet been reported by the end of that year. For Scotland and Northern Ireland death statistics are published by registrations. Death data for Scotland can be obtained from the General Register Office for Scotland (GROS). For Northern Ireland death data are available from the Northern Ireland Statistics and Research Agency (NISRA).

4.5.1.2 Annual death data - availability

Death data are available for download from the ONS website, but they are generally not available by single year of age for each calendar year. These data have to be obtained directly from the ONS.

According to “Historic Mortality Tables - 1837 to latest year and Notes to Tables”, the ONS takes a death registration extract about 3 months after the end of the year. Death occurrences are released with a 9-10 month lag and are not revised if new information comes to light subsequently, but may roll into the next year’s numbers if appropriate, e.g., very late reporting of deaths.

Death data by socio-economic grouping are also of potential interest to anyone trying to understand mortality, as mortality rates are different for the different groups. Data by socio-economic grouping is available through the ONS Longitudinal Study (LS), which surveys 1% of the population since the 1971 census. The groupings currently follow the Registrar General’s socio-economic classification:
Spouses not in employment are normally assumed to take their partners’ socio-economic classification.

Since the LS only has 1% coverage, it may not be very accurate in any given year, but it is suitable for determining trends of the population. (Class I group covers 5% of the population, so the number of people surveyed in Class I represents just 0.05% of the population). More details can be found on “Longitudinal Study 1971-2001: Completeness of Census Linkage”, September 2003.

4.5.1.3 Annual death data - sources of inaccuracy

Although deaths are accurately reported, reporting of age at death is less reliable, especially at higher ages. Where incomplete information is given on death registrations then approximations are made as appropriate. For example if only age at death is available rather than date of birth, then the date of birth is assumed to occur mid-year.

4.5.2 UK Population

4.5.2.1 Annual population data - collection

Population estimates for England & Wales are produced by the ONS Centre for Demography, who took over population forecasts from the Government Actuary’s Department (GAD) as of 31 January 2006. The latest population estimate (mid-2005) for England & Wales was made available in August 2006, some 14 months later. Population estimates relate to the mid-year position, so dividing deaths by this estimate produces the central rate of mortality.

Population data for England & Wales are subdivided by individual year of age and sex (gender) starting from 1961. From 1961 to 1970, individual years of age are available up to age 84 and then grouped from ages 85 and upwards. From 1971 onwards, individual years of age are available up to age 89 and then grouped from ages 90 and upwards.

Base population estimates are produced following each census, which takes place in the UK every ten years. The census provides a snapshot of the population in terms of gender, age, marital status and other characteristics such as employment and qualifications.

The most recent census took place in April 2001 and the census population estimate was projected to 30 June 2001 following revision for under-enumeration through a census coverage survey. Further revisions to the data followed in 2003 and 2004 based on local authority studies which picked up significant under-enumeration in Westminster and Manchester. Further details can be found in “2001 Census Local Authority Population Studies: Full Report”, September 2004, ONS.

4.5.2.2 Annual population data - availability

Population data going back to 1981 are readily available on the ONS website. Data are available at the UK and constituent country levels. At these levels, data are available by age and gender. Prior to 1981 data must be requested directly from the ONS.

Inter-censal estimates are the population estimates produced after the completion of each decennial census in respect of that census year. Although they are more accurate, inter-censal estimates are only available every ten years. For every year in between census years, the population estimates released are referred to as post-censal estimates.

4.5.2.3 Annual population data - sources of inaccuracy

The information gathered from each census should provide the most accurate snapshot of the population at that time. However there are sources of inaccuracy (i) because there will be people who should participate in the census but do not or (ii) because the information provided is incorrect, e.g., reporting of high ages may be less reliable. As a result some approximations will be needed to make up for these inaccuracies and one step in this process is to verify the sources of error.

On the basis of the 1991 Census, the forecast UK population for 2001 differed from the 2001 Census estimate by over a
million (or approximately a 2% error). In anticipation of such a forecast error, the ONS conducted a “One Number Census” in 2001, i.e., both a Census and a Census Coverage Survey were conducted. The conclusions of this investigation were that:

a) The 1991 population was overestimated by 350,000
b) There was a 200,000 under-enumeration in the 2001 Census
c) There was a restatement of migration between 1991 and 2001 by 300,000
d) There was an unexplained residual of 200,000


Following each census year, mid-year population estimates are rolled forwards year-by-year using three components of change: annual estimates of births, deaths and migrations. Births and deaths are considered to be nearly 100% accurate. However, migration figures are less precise and are only a best estimate.

International migration is based on the “International Passenger Survey” (IPS) of “intentions” on key transit routes, reported asylum seekers and Armed Forces movements. This estimates total annual immigrant inflow of about 500,000 and outflow of 300,000, with errors in the order of 3%. There are issues with accuracy due to lack of data on asylum seekers (these data come from Home Office as the IPS does not cover asylum seekers), land routes to Ireland and the reliability of a survey of intentions. For more details see “Estimating International Migration for Population Estimates: An Information Paper”, National Statistics Centre for Demography.

In addition there are data inaccuracies due to the steady stream of short-term migrants who are not covered by the survey, since their intended length of stay is less than 12 months. The UK also has a permanent population of seasonal migrants including those from many European Union “accession” countries. There are also potential problems with “visitor switchers” (short term visitors who subsequently decide to stay longer than a year) and “migrant switchers” (intended migrants who later decide to return home).

Emigration figures are even less accurate since the sample size surveyed can be very small at some ports. Information is not formally collected upon leaving the UK, so the data that are available could be just based on a few dozen interviews for example.

Migration within the UK is captured through National Health Service (NHS) registrations with doctors (general practitioners). This is considered accurate although there can be issues with time lag on registration, especially in some groups such as young males.

4.5.2.4 Population projections

In addition to population estimates, which relate to the recent or historic population, population projections for the next 70 years are produced by the ONS/GAD. The projections use assumptions concerning future mortality, fertility and migration, which are based on past trends, input from the registrars’ offices and other government offices.

Decennial population projections based on the census are produced every 10 years, for England, Wales, Scotland and Northern Ireland. Interim projections are produced every 2 years, with the most recent year being 2004. The projections from base year 2006 will be released in October 2007 (10 month lag, but the two-year cycle means that latest estimates can be up to 22 months out of date). See “2004-based National Population Projections for UK and Constituent Countries”, Spring 2006, Population Trends 123, National Statistics.

4.5.3 UK Life Tables

4.5.3.1 Decennial life tables

“English Life Tables” (ELTs) despite their name are graduated decennial tables for England & Wales based on the decennial censuses since 1911, apart from 1941. Prior to that their publication was less regular with the first table (ELT1) published in respect of the year 1841. See Forfar & Smith (1985-1987) for more details. The continued production of the English Life Tables beyond ELT16 (based on 2001
Census) is currently under review. Equivalent Scottish tables are available in 10-year intervals from 1911 to 1991.

4.5.3.2 Annual life tables
Life tables are also produced annually by the ONS (previously by the GAD). They are known as “Interim Life Tables” (ILTs) and are based on 3-year averages of crude deaths divided by mid-year population estimates. Data for each year are available from the ONS by single year of age and sex for the whole of the UK and its individual countries. These have been produced annually since 1982. Mortality rates are made available around the end of the following year, so Interim Life Tables in respect of 2005 were made available in late 2006. Unlike ELTs, ILTs are not graduated or smoothed across age. Graduation seeks, for example, to avoid having a low number of 64 year old deaths, but a high number of 65 year old deaths, followed by a low number of 66 year old deaths.

4.5.3.3 Older ages
At older ages mortality rates are less reliable because small errors in population size or number of deaths or reported age at death can have a substantial impact on the mortality rate estimates. There are potential problems with data records for some single elderly people living alone and this arises in all social classes. At very old ages it is hard to tell whether mortality rates stay at around 0.5 (as often assumed) or deteriorate exponentially.

Consequently the ONS cuts off raw mortality rates calculated above age 89 and groups all ages from age 90 and above. Prior to 1971, this cut-off point was made at age 84 rather than 89. See “National Population Projections: Review of Methodology for Projecting Mortality”, 2001, National Statistics Quality Review Series Report No 8.

For estimating mortality from ages 90 and above the ONS uses the method of “extinct generations” (see Appendix A). This uses death data available for the over 90s to get a more accurate mortality profile of each cohort aged 90 or more in the past. However one problem is that population estimates made by working forward from age 90 may not match those made by working backwards from age 120 to age 90 using this method.

4.5.3.4 Life table projections
Until the ONS Centre for Demography took over this function from the GAD in 2006, mortality projections had been produced by the GAD and revised every 2 years. In recent years the long-term projections have progressively made greater allowances of future mortality improvements. For the 2000 projections it was assumed that annual rates of mortality improvement would start to halve every 10 years from 2025 onwards. For the 2002 projections it was assumed that annual rates of improvement would halve every 25 years from 2027 onwards. For further details see “National Population Projections 2002-based”, 2004, Series PP2 No 24, National Statistics and GAD.

In the 2004 projections the view being held is that there is no reason for improvement rates to decline after 25 years. The result is that mortality improvements beyond 2029 years are currently assumed to be steady at 1% per year for all ages, male and female. For further details see “National Population Projections 2004-based”, 2006, Series PP2 No 52, National Statistics and GAD.

Variant improvement rates are used as stress tests and include:

- 0% after 25 years (low improvement case)
- 2% after 25 years (high improvement case)

The central annual improvement rate of 1% after 25 years is a reflection of the average annual improvement rate over the whole 20th century. The GAD assumes exponential convergence to the long term target improvement rates prior to 25 years: more rapidly than linear initially for males and less rapidly than linear initially for females.

Mortality improvements are assessed by calendar year for people born since 1960 and by cohort by people born prior to 1960. This is to address the “cohort effect”, where some birth year cohorts appear to exhibit higher rates of mortality improvement than others, e.g., cohorts born in 1930 and surrounding ages generally show higher than average rates of mortality improvement.

Although the methods used by the GAD are well documented, in practice it may not be possible to replicate
these projections exactly because not all the data that the GAD used may be available to the public.

4.6 Data for US

4.6.1 US Mortality

4.6.1.1 Annual death data - collection

Annual death data in the United States is compiled by the Centers for Disease Control and Prevention and the National Center for Health Statistics (CDC and NCHS). Death registration data have been collected for the entire United States since 1933. Prior to that, death statistics were only published for certain states and cities (CDC Vital statistics of US, 1999).

Death registrations are collected from the 50 States and the District of Columbia and are available with classifications such as geographic area, age, sex, and race. Before 1970, resident mortality statistics included all deaths occurring in the United States, including those of non-residents, which were coded according to place of death. For 1970 and later, deaths of non-residents were excluded from US resident mortality data.

Note that monthly data are also available, although these data are presented in aggregate and not split by age or sex. This information is generally available with about an eight-month lag.

4.6.1.2 Annual death data - availability

While death data are available for download from the CDC website, they are generally not available by single year of age for each calendar year, even in recent years. Instead, the CDC makes available, by CD-ROM, files for each calendar year called the “Public Use Data Files”. The files are also available on the website for the National Bureau of Economic Research and are listed as the “Vital Statistics NCHS’s Multiple Cause of Death Data”. These files are a listing of each registered death, along with information such as place of residence, place of occurrence, age, sex, race, cause of death, education, etc. As of this writing, these files are available for calendar years 1968-2004.

Final death data are currently released with approximately a three year lag. For example, 2003 final death data became available in April, 2006. Provisional data are available several months earlier, although not provided by single year of age. These deaths generally represent some large percentage of deaths for a calendar year, such as 90%.

4.6.1.3 Annual death data - sources of inaccuracy

Death data are believed to be quite complete because all States have adopted laws requiring the registration of deaths. The CDC believes that more than 99 percent of deaths occurring within the United States are registered (CDC Vital statistics of US, 1999).

Electronic copies of death records are collected from each of the 50 States and the District of Columbia. These records are audited for errors such as coding errors, key entry errors, and processing errors. The CDC estimates that errors occur in only 0.25% of the data overall.

Two other sources of error exist with respect to lack of available data. A small number of deaths are reported with unknown age. In 1999, there were 356 such resident deaths (0.01%). In the calculation of life tables, the CDC methodology is to distribute such deaths proportionately among age categories. Also, for 1972, death statistics were based on only a 50-percent sample of all deaths because of resource constraints.

Finally, sources of error exist with the misreporting of age at death (CDC Vital statistics of US, 1999). For example, age at death may result from the lack of birth certificate for the deceased, or from an estimate of death age from family members.

4.6.2 US Population

4.6.2.1 Annual population data - collection

Annual population estimates are developed by the U.S. Census Bureau as part of the Population Estimates Program (www.census.gov). This program develops total resident estimates of population as well as components of the changes in demographics each year, e.g., births, deaths, and migration. These estimates are developed on a monthly basis.
starting from population bases computed every tenth year as part of the decennial census. The U.S. census is mandated by the Constitution of the United States and has occurred every 10 years since 1790 (Siegel and Swanson, 2004).

To estimate annual populations from the base decennial census data, “post-censal” estimates are constructed. These estimates start with the most recently completed census and are adjusted monthly based on the components of population change:

- Births
- Deaths
- Net immigration
- Net movement of military personnel

These statistics are collected from the CDC and the Department of Defense. While post-censal estimates are updated monthly, the reference date for annual post-censal estimates used by the CDC is July 1. Each year, these estimates are revised for all previously released years using the most up-to-date demographic components (www.census.gov).

Once a formal census is conducted at the subsequent decennial, “inter-censal” estimates for the prior years are produced. These inter-censal estimates are adjusted from their original post-censal estimate values by the differences between the actual subsequent census and the post-censal estimate for that census.

4.6.2.2 Annual population data - availability

Significant amounts of population information are readily available on the website for the U.S. Census Bureau (www.census.gov). Data are available at national, state, and county levels. At these levels, data are generally available by age, gender, and race. National data are available on the website for each year since 1900, but state and county level data are only available on the website for years after 1969.

Post-censal estimates are released monthly, with approximately a three-month lag in reporting. Inter-censal estimates are released after the completion of the decennial census.

4.6.2.3 Annual population data - sources of inaccuracy

Enumeration errors in the latest decennial census, and consequently the post-censal estimates, impact the calculation of death rates. The sources of these errors are generally miscounting or from misreporting of age. These errors are often more serious for some subsets of the population, i.e., some age, race, and gender groups are more completely covered than others. For example, review of the 1990 census indicated that females were more completely counted than males, and the white population was more complete than the black population. By age, individuals 85 and over were the least accurate (CDC Vital statistics of US, 1999).

4.6.2.4 Population projections

The U.S. Census Bureau develops population projections with the Federal State Cooperative Program for Population Estimates (FSCPE). These are available at the state and national levels. Current practice is to produce national population projections every 3 to 4 years. The most recent population projection was based on the 2000 census, and projected population levels to 2050. These projections are available by age, gender, and race. The prior projection study was completed based on 1990 data and projected population levels through 2100 (www.census.gov).

4.6.3 US Life Tables

Vital statistics are published annually by the Centers for Disease Control and Prevention/National Center for Health Statistics. The NCHS publishes provisional monthly mortality statistics and final annual and decennial data.

4.6.3.1 Decennial life tables

The complete life tables for the U.S. population are based on decennial census populations and death data for the three-year period surrounding the census. These official decennial life tables are published by single year of age and have been prepared since 1900 (www.cdc.gov/nchs). The
tables present mortality rates and life expectancy by age (up to 110), by race, and by gender.

**4.6.3.2 Annual tables**

Annual tables have been published since 1945. They are published provisionally just after the close of the year and are published in final, revised form in the annual vital statistics reports (three-year lag). The provisional tables are published in 5-year age groups (abridged) and are based on a sample of deaths. The final annual tables have also traditionally been published in an abridged form. Prior to 1997, they were available in 5-year age buckets with the last group at 85 and older. Beginning in 1997, this methodology was changed to publish the final tables by single year of age, with the final age being 100 and older. These annual tables are based on annual death data and post-censal population data.

**4.6.3.3 Older ages**

As described above, mortality rates at older ages have only recently been published by the CDC. There were several reasons for this change. First, the CDC noted that survival rates beyond age 85 were increasing. Second, data from sources other than the vital statistics are available and credible (Method for Constructing Complete Annual U.S. Life Tables, 1999). For these reasons, the CDC began to use Medicare data to develop mortality rates for ages 85 and older, as this data is known to be more reliable than those based on vital statistics (National Vital Statistics Reports, 2006).

**4.6.3.4 Life table projections**

The Social Security Administration, provider the of Old-Age, Survivors, and Disability Insurance (OASDI) programs, prepares life table projections for the purpose of estimating the future financial health of the programs (www.ssa.gov/OACT/NOTES/s2000s.htm). The most recent study, published in August 2005, presented mortality projections based on the 2000 decennial census and Medicare data projected to 2100. These projections were developed on the basis of reductions in ultimate mortality split by seven separate cause-of-death categories:

1. Heart Disease
2. Cancer
3. Vascular Disease
4. Violence
5. Respiratory Disease
6. Diabetes Mellitus
7. All Other Causes

The study projected three scenarios representing high, intermediate, and low estimates of future mortality rates. The intermediate projections reflect the SSA’s best estimate of future experience (Bell and Miller, 2005).
Part II
LifeMetrics Index
5. Definition of LifeMetrics Index

5.1 Introduction

The LifeMetrics Index is based on publicly available mortality data for national populations, broken down by country, age and gender. Initially, the LifeMetrics Index includes data for the US and for England & Wales.

The methods and algorithms used in the construction of the LifeMetrics Index were chosen to be as objective as possible, without sacrificing transparency or the integrity of the underlying data. The data sources, methodologies, algorithms and calculations used in the development and production of the index are fully disclosed in this Chapter and in an Appendix to this document containing a full technical description.

5.1.1 Index metrics

The LifeMetrics Index consists of a number of components which measure longevity and mortality in differing but complementary ways. The principal components of the index are:

- Crude central rate of mortality ($m_x$)
- Graduated initial rate of mortality ($q_x$)
- Period life expectancy ($e_x$)

We classify index data in terms of population and gender and with reference to a specified age group and time period. In full, a data point can be referred to as

Data(Population, Gender, Age group; Period)

For instance, a mortality rate, $m$, might refer to data for England & Wales, to the male population, and be based on underlying data for those aged 65 in the year 1992. We would refer to this particular piece of data as

$m$ (England & Wales, Male, 65; 1992).

We have chosen the order of notation such that in typical use, subsets of data are likely to involve specifying the latter terms relative to those earlier in the precedence. In practice we will often shorten the description where the application to a particular item of reference is clear. The notation is designed to be used consistently beyond the current bounds of published index data.

5.2 Defining the mortality metrics used in the index

5.2.1 Crude central rate of mortality, $m_x$

The central rate of mortality can be considered as the base measurable mortality rate for a dataset, since raw exposure data is typically available by mid-year population. The rate is calculated as the number of deaths in a given year for a given age ($D_{x,t}$) divided by the mid-year population estimate in the year for that age (also called the central exposed-to-risk, $E_{x,t}$, i.e.,

$$m_x = \frac{D_{x,t}}{E_{x,t}}$$

In the LifeMetrics Index, we publish the "crude" central rate of mortality for ages 20 to 89 inclusive for England & Wales (up to age 84 before 1971), and ages 20 to 84 inclusive for the United States. Beyond these ages, raw data by individual age is currently unavailable or considered to be unreliable due to difficulty in recording high age data accurately.

The terminology "raw" or "crude" is generally used to indicate that the calculation is run directly on the data for number of deaths and exposure population as published, before any smoothing, averaging or adjustment by the relevant national agency has taken place.

In Tables 5.1 and 5.2, we show the raw central rate of mortality for England & Wales males and females respectively, for the current reference year, 2005. In Tables 5.3 and 5.4, we show the raw central rate of mortality for United States males and females respectively, for the current reference year, 2004.

5.2.2 Graduated initial rate of mortality, $q_x$

Mortality tables used for the estimation of future, uncertain longevity-linked cash flows and pricing/valuation of pensions and life insurance liabilities are typically quoted...
in terms of a "smoothed" or "graduated" initial rate of mortality.

In the LifeMetrics Index, we publish graduated initial rates of mortality for ages 20-90 inclusive for both England & Wales and the United States. By graduated, we mean that we have applied a smoothing methodology to eliminate noise and errant data points from the raw data for $m_x$ to create a stable set of $q_x$ which are generally monotonic with age (although this is not applied as a condition) and representative of the "true" underlying mortality rates for the period.

We use a cubic-spline graduation algorithm applied to the natural logarithm of the central mortality rates for this purpose. Cubic splines are well known in the field of interest rates and have become an increasingly popular methodology for graduation since the introduction of cheap and powerful computers. They have been used in the UK by the Government Actuary’s Department (GAD) for the production of the last few decennial English Life Tables based on national mortality data. Beyond the range of the raw central mortality rate data, we linearly extrapolate the final cubic spline to age 90. In particular, for England & Wales we extrapolate from age 89 to age 90, and for the US from age 84 to age 90.

We apply an algebraic approximation to these smoothed $m_x$ to generate the final smoothed $q_x$. The approximation used to generate these initial rates of mortality from the central rates of mortality is:

$$ q_x = \frac{m_x}{1 + \frac{1}{2} m_x} $$

A full description of the graduation, extrapolation and transformation methodologies is contained in Appendix A.

In Tables 5.5 and 5.6, we show the graduated initial rate of mortality for England & Wales males and females respectively, for the current reference year, 2005. In Tables 5.7 and 5.8, we show the graduated initial rate of mortality for United States males and females respectively, for the current reference year, 2004.

To illustrate the importance of graduation, in Figure 5.1 we plot "raw" data for the central mortality rate along with the "smoothed" data for the central mortality rate after graduation for England & Wales males for reference year 2005. We have utilised a logarithmic scale for the mortality rate axis to make the mortality rates more comparable graphically across ages. It is clear that relative to the raw data, graduation has removed the noise from age-to-age mortality rates and created a mortality curve which has an intuitive shape under natural demographic and biological assumptions.

5.2.3 Period life expectancy, $e_x$

Period life expectancy gives an indication of the average life expectancy at a given age for a given period mortality table without allowing for future improvements in mortality rates. It is a useful benchmark with which to compare mortality tables from different periods in order to capture how
### Table 5.1 Crude central mortality rates for England & Wales males, reference year 2005

<table>
<thead>
<tr>
<th>Age (x)</th>
<th>Central rate of mortality, m_x (%) (England &amp; Wales, Males; 2005)</th>
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</thead>
<tbody>
<tr>
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### Table 5.2 Crude central mortality rates for England & Wales females, reference year 2005

<table>
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<tr>
<th>Age (x)</th>
<th>Central rate of mortality, m_x (%) (England &amp; Wales, Females; 2005)</th>
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### Table 5.3 Crude central mortality rates for United States males, reference year 2004

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<th>Central rate of mortality, m_x (%) (United States, Males; 2004)</th>
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### Table 5.4 Crude central mortality rates for United States females, reference year 2004

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March 13, 2007

LifeMetrics: A toolkit for measuring and managing longevity and mortality risks
Table 5.5 Initial mortality rates for England & Wales males, reference year 2005

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Table 5.7 Initial mortality rates for United States males, reference year 2004

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Table 5.8 Initial mortality rates for United States females, reference year 2004

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Table 5.9 Period life expectancy for England & Wales males, reference year 2005

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Table 5.11 Period life expectancy for United States males, reference year 2004

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<td>43.26</td>
<td>63</td>
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<td>39</td>
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<td>41</td>
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<td>66</td>
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<td>42</td>
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</tr>
<tr>
<td>43</td>
<td>38.55</td>
<td>68</td>
<td>17.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>37.63</td>
<td>69</td>
<td>16.43</td>
<td></td>
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</tr>
</tbody>
</table>
changes in mortality rates impact overall life expectancy without the interference of improvement assumptions on which practitioners tend to be less in agreement.

In terms of the underlying $q_x$ in the index, period life expectancy can be calculated by

$$e_x = \sum_{i=1}^{j} \left( \prod_{j=0}^{i-1} (1-q_{x+j}) \right)$$

In the LifeMetrics Index, we publish period life expectancy for ages from 20 to 80 inclusive. These are based on the smoothed initial mortality rates, $q_x$, as published up to age 90. Beyond age 90, where we have not explicitly calculated mortality rates, we follow a common approach and fit an appropriate parametric shape to high age mortality rates. We do not directly publish mortality rates beyond age 90 since there is a fair degree of uncertainty in their estimation. We note that the impact these rates have on uncertainty of life expectancy is far less significant.

For consistency with our graduation methodology, we fit a single cubic polynomial between ages 90 and 119. We then close out the mortality table by assuming a mortality rate of 100% at age 120 (again a common practice). A full description of the high age methodology is contained in Appendix A.

In Tables 5.9 and 5.10, we show the period life expectancy for England & Wales males and females respectively, for the current reference year, 2005. In Tables 5.11 and 5.12, we show the period life expectancy for United States males and females respectively, for the current reference year, 2004.

### 5.3 Data sources and timeline

The raw data required to evaluate the index, for a given country, gender and age, can be broken down into:

- The size of the population in question (the exposure to risk)
- The number of deaths occurring within that population in a given period (typically 1 year)

#### 5.3.1 England & Wales

Data for mid-year population estimates are obtained from the Population Estimates Unit of the UK Office for National Statistics (ONS). Data for a given year are typically published in August of the following year. However there can be revisions to the population estimates after they are first published.

The historic population data used in the index for reference years 1961-2004 are population estimates as of 14 August 2006. For the current reference year (2005) the population data used in the index relates to the population estimate as of 14 February 2007. In future years the index calculation will use the latest population estimate for the reference year as of the final data release for deaths for that year.

Data for deaths are obtained from the UK Office for National Statistics (ONS). We note that data are available by occurrence or registration. Final data by occurrence for a given year are typically published in September/October of the following year and are not revised. From reference year 2005 onwards, death data used in the index relates to this final occurrence data as published by the ONS. It will not be revised retrospectively using registration data released in 2007 or subsequent years.

Historic death data for England & Wales, up to and including reference year 2004, relate to occurrences reconstructed from registration data up to, and including, reference year 2005. This is because, looking back historically, registration data have been a more accurate source in that they include information that has become available after the year in which the deaths actually occurred. This is best illustrated with an example. In order to estimate the number of deaths that occurred in 2003, one could look at the final occurrence data published in say October 2004. Alternatively one could take the registration data from 2003 onwards and piece together the deaths occurring in 2003 that were registered in 2003 and each subsequent year. The vast majority of deaths occurring in 2003 will be picked up in the registration file for 2003 published in spring 2004. However, additional deaths will have been picked up in the registration file for 2004 published in spring 2005 and this
will include some deaths that had not been recorded in the occurrence data published in autumn 2004. Similarly, the registration file for 2005 will contain further deaths that occurred in 2003. Therefore summing the deaths occurring in 2003 that are registered in the years 2003, 2004 and 2005 will provide a more complete death count than using the ‘final’ 2003 occurrences reported. We use this method to reconstruct death data up to reference year 2004 inclusive.

5.3.2 United States

Annual population estimates are obtained from the US Census Bureau and are based on mid-year estimates (July 1 reference). For consistency with the number of deaths included in the calculation of mortality rates, population data refer to residents only. Listed here are the specific data files used for index construction:

- Data from 1968-1989 are based on “inter-censal” estimates
- After 1989, all population data are based on “post-censal” estimates

Post-censal estimates are released monthly, with approximately a three-month lag in reporting. Note that data for a given year are periodically revised until the processing of the following census has been completed. For index calculation, we use the latest population estimate for the reference year as of the final data release for deaths for that year. Data for 1968-2003 were collected on 30 August 2006. Data for 2004 were collected on 20 February 2007.

Annual death data by gender and single year of age is obtained from the US Centers for Disease Control and Prevention (CDC)/National Center for Health Statistics (NCHS). Final death data for index construction can be obtained either from the Public Use Data File (NCHS - on CD-ROM) or the Multiple Cause of Death Data Files (National Bureau of Economic Research - www.nber.org/data/multicause.html). These files contain individual records of death with information on place of residence, occurrence, age, sex, race, cause of death, education, etc. For consistency with the CDC’s Annual Life Tables, during the construction of the index the following records are excluded from the Public Use Data File/Multiple Cause of Death Data File:

- Deaths for non-residents, i.e., only include deaths registered in the 50 States and the District of Columbia
- Deaths without reported age

Note that prior to 1970, death data included all deaths occurring in the United States, and therefore deaths of non-residents are included in these statistics.

The CDC makes available this final death data with a lag of approximately two to two and a half years. For example, 2004 death data became available in November 2006.

5.4 Index Governance

The LifeMetrics Index is maintained by JPMorgan and governed by the LifeMetrics Index Advisory Committee. Members of the Advisory Committee are drawn from JPMorgan, experienced consultants, leading academics and industry practitioners.

The responsibilities of the Index Advisory Committee include reviewing the methodology and data used in determining the index, providing guidance for changes in methodology where appropriate, and providing guidance on new indices that may be added in the future.

The Index Advisory Committee meets at least twice a year, to review the data and calculations made by the Calculation Agent, verify the methodology and deal with other issues that may arise.

5.5 Index Distribution

5.5.1 Production

The latest series for the LifeMetrics Index will be released annually. The publication date is a function of raw data availability, and will take place shortly after the release of all required raw data by the reporting agencies for each particular index.

For a given reference year, raw data is generally published by the relevant government agencies with a time lag which
can number into multiple years. Therefore the LifeMetrics Index data published in a given year (“Publication Year”) will be based on source data that references a specified prior year (“Reference Year”). The time lag will differ depending on the population geography.

The release date for the latest series will be announced on the LifeMetrics website prior to the actual release date.

For England & Wales, data for a new reference year is expected to be published between 10 and 12 months following the end of the reference year.

For the United States, data for a new reference year is expected to be published between 22 and 30 months following the end of the reference year.

These timelines are based on the expectation that raw data will be released by the relevant agencies within the following timeframes:

England & Wales
- Death data: Typically October of the year following the reference year
- Population data: Will use latest revision as of release of death data above

United States
- Death data: Recently, data has been released between November of the second year and April of the third year following the reference year
- Population data: Will use latest revision as of release of death data above

5.5.2 Availability

5.5.2.1 Website

The full set of index values, both current and historical, can be obtained from the LifeMetrics website at

http://www.jpmorgan.com/lifemetrics

Historical data is provided on the website going back to 1961 for England & Wales and 1968 for the United States. A summary of the available data is shown in Table 5.13.

5.5.2.2 Bloomberg

LifeMetrics data are available from Bloomberg at LFMT <GO>.

A number of sub-indices for the LifeMetrics Index series are available through unique tickers. The tickers are organized along the following logical construct:

LM{Data Type}{Gender}{Country Code}{Age} <INDEX>

For example, LMQMUS65 <INDEX> provides the value for the graduated initial mortality rate ($q_x$) for males in the United States aged 65.

The set of sub-indices currently listed on Bloomberg is shown in Table 5.14.
### Table 5.13 LifeMetrics Index data available on the website

<table>
<thead>
<tr>
<th>Data type</th>
<th>Gender</th>
<th>Country code</th>
<th>Age</th>
<th>Period</th>
</tr>
</thead>
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<td>M / F</td>
<td>US</td>
<td>20 – 84</td>
<td>1968 – 2004</td>
</tr>
<tr>
<td>M</td>
<td>M / F</td>
<td>EW</td>
<td>20 – 89</td>
<td>1971 – 2005</td>
</tr>
<tr>
<td>M</td>
<td>M / F</td>
<td>EW</td>
<td>20 – 84</td>
<td>1961 – 1970</td>
</tr>
<tr>
<td>Q</td>
<td>M / F</td>
<td>US</td>
<td>20 – 90</td>
<td>1968 – 2004</td>
</tr>
<tr>
<td>Q</td>
<td>M / F</td>
<td>EW</td>
<td>20 – 90</td>
<td>1961 – 2005</td>
</tr>
<tr>
<td>E</td>
<td>M / F</td>
<td>EW</td>
<td>20 – 80</td>
<td>1961 – 2005</td>
</tr>
</tbody>
</table>

M = Crude central mortality rates  
Q = Graduated initial mortality rates  
E = Period life expectancy  

Individual ages for which index values are available

#### Table 5.14 LifeMetrics Index - sub-indices listed on Bloomberg

<table>
<thead>
<tr>
<th>Data type</th>
<th>Gender</th>
<th>Country code</th>
<th>Age</th>
</tr>
</thead>
<tbody>
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<td>EW</td>
</tr>
<tr>
<td>LM</td>
<td>Q</td>
<td>M / F</td>
<td>US / EW</td>
</tr>
<tr>
<td>LM</td>
<td>E</td>
<td>M / F</td>
<td>US / EW</td>
</tr>
</tbody>
</table>

M = Crude central mortality rates  
Q = Graduated initial mortality rates  
E = Life expectancy  

Individual ages for which index values are available
6. Historic index data

6.1 Introduction

In this chapter we examine the historical evolution of mortality for England & Wales (EW) and the United States (US), in terms of the historical LifeMetrics Index data. This chapter should also provide the reader with a starting point for analyzing historical mortality data from any source.

We focus on the results for the graduated initial rates of mortality, because these are considered to be closer to the “true” underlying mortality rates than the crude central rates of mortality from which they are derived. We also examine period life expectancy as a benchmark of how mortality rates have changed over time. Finally, we end the chapter with a comparison of mortality experience from England & Wales and the United States.

As an introduction to analyzing specific mortality data, it is useful to first define the concept of the “mortality curve”. In broad terms, a mortality curve is used to describe a graph of mortality rate plotted against age, and it provides a snapshot of the level of mortality at a given point in time. In some respects the mortality curve is analogous to the yield curve in the context of interest rates, where yield is plotted against term to maturity. An example of a mortality curve is presented in Figure 6.1.

Because small changes in mortality are often difficult to view graphically using a standard mortality curve, figures are typically presented on a logarithmic scale. The transformation of the mortality curve from Figure 6.1 to a logarithmic scale is shown in Figure 6.2. This transformation clearly makes the mortality rates more comparable graphically across the full range of ages.

6.2 Analysis for England & Wales

6.2.1 Graduated initial mortality rates

In Figures 6.3 and 6.4, we plot the graduated initial mortality rates ($q_x$) from ages 40 to 90 for England & Wales males and females, for reference years at ten year intervals from 1961 to 2001. These figures illustrate how the mortality curve has changed through time across ages. On a logarithmic scale, mortality rates are approximately linear and have experienced parallel shifts downwards through time.

In Figure 6.5, we show the cumulative improvement in initial mortality rate by age between 1961 and 2005 for England & Wales males and females. For males, the cumulative improvement can be as low as 35% but between the ages of 55 and 65, the improvement is consistently higher than 57%. By comparison, female improvements are in general more uniform across ages. To illustrate the effect of graduation, Figure 6.6 shows the same calculation for the crude central mortality rates. It is clear that graduation has established a smooth relationship between improvements across age.
Figure 6.3 EW males, initial rate of mortality curve

Figure 6.4 EW females, initial rate of mortality curve

Figure 6.5 EW graduated initial mortality rate improvements for the period 1961-2005

Figure 6.6 EW crude central mortality rate improvements for the period 1961-2005

Table 6.1 Volatilities of graduated initial and crude central mortality rate improvements for England & Wales males and females ages 45, 55, 65, and 75

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Graduated $q_x$</td>
<td>Crude $m_x$</td>
</tr>
<tr>
<td>45</td>
<td>2.96%</td>
<td>6.16%</td>
</tr>
<tr>
<td>55</td>
<td>2.57%</td>
<td>3.70%</td>
</tr>
<tr>
<td>65</td>
<td>2.64%</td>
<td>2.84%</td>
</tr>
<tr>
<td>75</td>
<td>3.03%</td>
<td>3.62%</td>
</tr>
</tbody>
</table>
6.2.1.1 Historical index levels

Figures 6.7 - 6.10 highlight the evolution of the LifeMetrics Index for these initial mortality rates through time for the period 1961-2005 for ages 45, 55, 65, and 75 respectively.

We note that males have generally shown larger improvements than females. However, at age 45, the male initial mortality rate has decreased from 0.42% in 1961 to 0.23% in 2005, representing an improvement of 45%; while for the same period the initial mortality rate for a female age 45 showed an improvement of 48%. The improvements were 58%, 59% and 53% for males age 55, 65, and 75 respectively; and 44%, 48% and 51% for females age 55, 65, and 75.

6.2.1.2 Volatility of mortality rates

The previous figures illustrate how mortality rates have demonstrated a clear trend through time. On the other hand, the year-on-year improvements show much more variability. For the same ages, these are shown in Figures 6.11 - 6.14.

The volatilities of the year-on-year improvements for ages 45, 55, 65, and 75 are relatively steady across ages and shown in Table 6.1. Here, volatility is defined as the standard deviation of year-on-year changes in the mortality rate. For comparison, we also show the crude central mortality rate improvement figures and note that the volatility of these improvements are, in general, significantly higher.

6.2.1.3 Autocorrelation

The graphical year-on-year improvements appear to show that years of mortality improvement are often followed by years of deterioration in mortality. This type of behaviour can be measured using autocorrelation. In Table 6.2, we show the autocorrelations for the year-on-year mortality improvements for selected ages over the period 1961-2005. The negative autocorrelations indicate that there is a definite tendency for upward swings in improvements to be followed by deteriorations.

6.2.1.4 Heat maps of mortality improvements

An alternative method for illustrating the data is to plot heat maps for the 1-year mortality improvements, as shown in Figures 6.15 and 6.16. These give a summary of how mortality has been improving at different ages and in different years and are effectively a graphical representation of a cohort life table, being closely related to the Lexis diagram seen in Chapter 3.

Cool colors, like blues, indicate that mortality is deteriorating (increasing), while warm colors like yellow and red indicate improvements (mortality rates are decreasing). The improvement rates have been smoothed through time (by taking the average over 5 years) to highlight the general trends in the data.

One interesting and well-documented feature of mortality rates is the cohort effect, which can be seen through the strong diagonals of similar colours in the heat map. In particular, cohorts born around the 1930’s have exhibited strong rates of improvements between ages 40 to 70 relative to cohorts born in the 1920’s or 1940’s.

6.2.2 Period life expectancy

In Figure 6.17, we show the evolution of period life expectancy over the period 1961 - 2005, for 65 year-old males and females in England & Wales. There has been a steady increase in life expectancy over the period with male 65 year old life expectancy increasing by 5.19 years and female life expectancy increasing by 4.62 years. Note that this information may also be presented graphically as a total life span by adding 65 years to the life expectancy at age 65 (as shown in Figure 6.18).

In Figure 6.19, we summarise the changes in period life expectancy from 1961 to 2005 for males and females at ages 45, 55, 65, and 75. This bar chart shows that life expectancies for males have generally increased more over the period than for females and that there have been greater life expectancy increases at younger ages.

6.3 Analysis for United States

6.3.1 Graduated initial mortality rates

In Figures 6.20 and 6.21, we plot the graduated initial mortality rates ($q_x$) from ages 40 to 90 for United States males
Figure 6.7 Age 45, EW mortality rates (1961-2005)

Figure 6.9 Age 65, EW mortality rates (1961-2005)

Figure 6.11 Age 45, EW mortality improvements

Figure 6.13 Age 65, EW mortality improvements

Figure 6.8 Age 55, EW mortality rates (1961-2005)

Figure 6.10 Age 75, EW mortality rates (1961-2005)

Figure 6.12 Age 55, EW mortality improvements

Figure 6.14 Age 75, EW mortality improvements
Table 6.2 Autocorrelation of graduated initial and crude central mortality rate improvements for England & Wales males and females ages 45, 55, 65, and 75

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<tr>
<td></td>
<td>Graduated $q_x$</td>
<td>Crude $m_x$</td>
</tr>
<tr>
<td>45</td>
<td>-14%</td>
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<tr>
<td>55</td>
<td>-16%</td>
<td>-31%</td>
</tr>
<tr>
<td>65</td>
<td>-23%</td>
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<tr>
<td>75</td>
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<td>-25%</td>
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</table>
and females, for reference years from 1968 to 2001 at ten year intervals (except for the first interval). These figures illustrate how the mortality curve has changed through time across ages. On a logarithmic scale, mortality rates are approximately linear and have experienced parallel shifts downwards through time.

In Figure 6.22, we show the cumulative improvement in initial mortality rate by age between 1968 and 2004 for United States males and females. For males, the greatest improvement (over 50%) occurs for ages in the range 56-66, while for females, improvements are in general more uniform across ages. To illustrate the effect of graduation, Figure 6.23 shows the same calculation for the crude central mortality rates. It is clear that graduation has established a smooth relationship between improvements across age.

6.3.1.1 Historical index levels

Figures 6.24-6.27 highlight the evolution of these initial mortality rates through time for the period 1968-2004 for ages 45, 55, 65, and 75 respectively.

We note that males have shown larger improvements than females. For instance, at age 45, the male initial mortality rate has decreased from 0.63% in 1968 to 0.37% in 2004, representing an improvement of 41%; while for the same period the initial mortality rate for a female age 45 showed an improvement of 39%. The improvements were 50%, 51%, and 43% for males aged 55, 65, and 75, respectively; and 40%, 34%, and 37% for females aged 55, 65, and 75, respectively.

6.3.1.2 Volatility of mortality rates

The previous figures illustrate how mortality rates have demonstrated a clear trend through time. On the other hand, the year on year improvements show much more variability. For the same ages, these are shown in Figures 6.28-6.31.

The volatilities of the year-on-year improvements for ages 45, 55, 65, and 75 are shown in Table 6.3. We note that volatility is significantly higher at age 45. Here, volatility is defined as the standard deviation of year-on-year changes in the mortality rate. For comparison, we also show the crude central mortality rate improvement figures and note that the volatility of these improvements are, in general, significantly higher.

Figure 6.17 EW period life expectancy for age 65

Figure 6.18 EW period life span for age 65

Figure 6.19 EW change in life expectancy from 1961 to 2005 for ages 45, 55, 65 and 75
Table 6.3 Volatilities of graduated initial and crude central mortality rate improvements for United States males and females ages 45, 55, 65, and 75

<table>
<thead>
<tr>
<th>Age</th>
<th>Graduated $q_x$ (1968-2004)</th>
<th>Crude $m_x$</th>
<th>Graduated $q_x$ (1968-2004)</th>
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<td>2.31%</td>
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<td>2.41%</td>
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</tr>
<tr>
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<td>1.53%</td>
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</tr>
<tr>
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<td>1.52%</td>
<td>2.42%</td>
</tr>
<tr>
<td>75</td>
<td>1.47%</td>
<td>2.70%</td>
<td>1.66%</td>
<td>2.60%</td>
</tr>
</tbody>
</table>
Figure 6.24 Age 45, US mortality rates (1968-2004)

Figure 6.25 Age 55, US mortality rates (1968-2004)

Figure 6.26 Age 65, US mortality rates (1968-2004)

Figure 6.27 Age 75, US mortality rates (1968-2004)

Figure 6.28 Age 45, US mortality improvements

Figure 6.29 Age 55, US mortality improvements

Figure 6.30 Age 65, US mortality improvements

Figure 6.31 Age 75, US mortality improvements
Table 6.4 Autocorrelation of graduated initial and crude central mortality rate improvements for United States males and females ages 45, 55, 65, and 75

<table>
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<tr>
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<tbody>
<tr>
<td></td>
<td>Graduated $q_x$</td>
<td>Crude $m_x$</td>
</tr>
<tr>
<td>45</td>
<td>43%</td>
<td>-21%</td>
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<tr>
<td>55</td>
<td>-9%</td>
<td>-42%</td>
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<tr>
<td>65</td>
<td>-12%</td>
<td>-30%</td>
</tr>
<tr>
<td>75</td>
<td>-18%</td>
<td>-37%</td>
</tr>
</tbody>
</table>
6.3.1.3 Autocorrelation

The graphical year-on-year improvements appear to show that years of mortality improvement are often followed by years of deterioration in mortality. This type of behaviour can be measured using autocorrelation. In Table 6.4, we show the autocorrelations for the year-on-year mortality improvements for selected ages over the period 1968-2004. The negative autocorrelations indicate that, at high ages, there is a definite tendency for upward swings in improvements to be followed by deteriorations.

6.3.1.4 Heat maps of mortality improvements

An alternative method for illustrating the data is to plot heat maps for the 1-year mortality improvements, as shown in Figures 6.32 and 6.33. These give a summary of how mortality has been improving at different ages and in different years and are effectively a graphical representation of a cohort life table, being closely related to the Lexis diagram seen in Chapter 3.

Cool colors, like blues, indicate that mortality is deteriorating (increasing), while warm colors like yellow and red indicate improvements (mortality rates are decreasing). The improvement rates have been smoothed through time (by taking the average over 5 years) to highlight the general trends in the data.

Although not as distinct as seen in the England & Wales data, the United States data, particularly females, display some small cohort effects.

6.3.2 Period life expectancy

In Figure 6.34, we show the evolution of period life expectancy over the period 1968-2004, for 65 year-old males and females in United States. There has been a steady increase in life expectancy over the period with male 65 year old life expectancy increasing by 4.32 years and female life expectancy increasing by 3.48 years. Note that this information may also be presented graphically as a total life span by adding 65 years to the life expectancy at age 65 (as shown in Figure 6.35).

In Figure 6.36, we summarize the changes in period life expectancy from 1968 to 2004 for males and females at ages 45, 55, 65, and 75. This bar chart shows that life expectan-
cies for males have generally increased more over the pe-
riod than for females and that there have been greater life
expectancy increases at younger ages.

6.4 Summary

It is clear from the analysis presented that mortality rates
have experienced notable improvements in both the United
States and England & Wales over the last three to four dec-
ades.

We end this chapter with a summary of the key points of
comparison and contrast between the England & Wales and
United States historical mortality experience.

Based on the data analysis, we conclude that there have,
in general, been:

• Higher rates of improvement in England & Wales than in
  United States
• Higher rates of improvement for males than for females
• Lower volatility of improvements in the United States than
  in England & Wales
• Stronger cohort effects in England & Wales than in United
  States
Part III
LifeMetrics Framework
7. Modeling future longevity and mortality

7.1 The big debate

The topic of estimating future levels of mortality has received much attention lately. There are many different opinions regarding how long people will live in the future. Some feel that lifespans will continue to increase at least as rapidly as experienced over the last five or six decades. Others feel that the increases in lifetime will decelerate, or potentially even decline, for certain groups.

Some of the debate surrounding forecasts of future mortality relates to whether decreases in mortality that have occurred in the past could occur again in the future. For example, in the US, life expectancy at birth has increased over the last century due primarily to decreases in mortality rates at younger ages. Some academics, like Jay Olshansky, argue that since such mortality decline has already occurred, it can not happen again. Therefore, any increase in longevity would have to occur by virtue of declines in mortality for other age groups, or for other underlying causes. It is also more difficult for life expectancy to keep increasing as it reaches higher levels (or, equivalently, for mortality rates to keep decreasing as they reach lower levels).

Other contentious points in the debate on forecasting mortality are (i) the possible existence of a biological limit to life and (ii) whether humans are approaching this limit. Some academics have argued that there exists a biological limit to life, and James Vaupel has traced this theory back as far as Aristotle. Vaupel argues, however, that life expectancy is not approaching a limit. Instead, he suggests that survival can be extended by various genetic changes and non-genetic interactions. Further discussion of all of these ideas is contained in Watson Wyatt (2005).

The debate on forecasting mortality also includes whether and how medical advances should be taken into account. While most forecasters agree that medicine has and will continue to reduce mortality (especially related to certain causes of death), there is disagreement as to whether advancements that, as of today, do not exist and might not happen should be incorporated into forecasts. Some forecasters project the development of technologies that could significantly delay aging and increase lifespan. Even if the forecaster believes that future medical enhancements should be included, it is often unclear how the impact of these types of enhancements might be reasonably estimated. See Willets (1999) for further information.

Figure 7.1, compiled from information in a 2005 Social Security Bulletin (Waldron, 2005), shows the range of US period life expectancy projections for results published using various models since 1992. We note that even at a 20 year horizon, there is significant discrepancy between forecasts.

These disparate beliefs about the change in lifespans, biological limits to life and medical advancement all beg the question of how to appropriately model future mortality. To some extent, this can be distilled into a question of whether a forecasting model should rely on statistical analysis alone or expert opinion based in biology. In the following section, a variety of mortality forecasting methods are explored.

7.2 Forecasting methods

In concert with the significant debate over future mortality, there are a number of broad approaches to forecasting mortality rates in current use. These approaches can be categorized in different ways. One way would be to classify them into:

Figure 7.1 US unisex period life expectancy in selected projection years, by study.
(i) Extrapolative models
(ii) Causal models involving econometric relationships
(iii) Models based on underlying biomedical processes

In the following section, we summarize some of the most commonly used methods in order of increasing reliance on expert opinion.

7.2.1 Extrapolation: deterministic & stochastic

An extrapolative model calculates estimates of future mortality using the current level of mortality and an estimate of the rate of change in future mortality. The rate of change in future mortality is based upon the changes observed in the recent to medium-term past. This observed trend in mortality changes is assumed to continue over the forecast horizon. Extrapolative models can be either deterministic or stochastic. The most well-known example of a stochastic extrapolative model is the Lee-Carter (1992) mortality projection model, described in section 7.3.1.1.

While this approach is sometimes thought of as completely objective, it does rely to some extent on a subjective judgment. It is based on the opinion that future mortality will continue to improve at the same rate as in the past. Additionally, the forecaster must choose the historic period over which mortality improvement rates should be analyzed to serve as the basis for forecasting.

Jay Olshansky presents several reasons why extrapolation methods should be used with caution (Olshansky, 1988). These relate to uncertainty surrounding the cause of recent mortality changes, possible impacts of a biological limit to life, future medical advances, environmental risk factors, and reliability of base mortality data. Since these concerns are less significant over short time periods, these models are more appropriate for short-term forecasts.

7.2.2 Cause-specific extrapolation

Cause-specific models disaggregate total mortality by cause of death and forecast mortality rates for each cause separately. The forecasts for each cause then involve extrapolating past trends. This can be more difficult than forecasting aggregate mortality rates since data available by cause are generally less reliable than aggregate data.

The advantage of the approach is that it can give a better understanding of the factors behind overall changes in mortality. However, accuracy of the resulting mortality forecasts is not always improved. In addition, one unresolved issue is that correlations between causes of death are generally not quantified.

7.2.3 Projection using relational models

One way of using relational projection models can be by reference to a more "advanced" population. In this approach, future mortality rates are assumed to follow the dynamics of observed mortality for a separate "target" population. In other words, the mortality profile of the target population is believed to be the mortality profile that the forecasted population will achieve over some future time horizon. Generally, this is applied under the assumption that mortality forecasts are considered likely to trend toward the mortality already observed in a longer-lived population. This approach, therefore, is more applicable to developing rather than developed countries.

7.2.4 Extrapolation with expert opinion

This approach is based on the extrapolation model, but explicitly includes strong assumptions by the forecaster in respect of future mortality. Clearly, using any mathematical model blindly without judgment and without an understanding of the context can be dangerous. Expert opinion can be an important factor in ensuring that a model is not pushed beyond its limits of validity. This explicit expert opinion could be as simple as an assumption about the ultimate level of mortality, or that mortality improvements follow a specified path through time. Note that the approach can be applied in terms of the aggregate mortality rates or by specific cause of death.

7.2.5 Cause-delay

The cause-delay approach assumes that improving mortality for a specific cause has an impact in delaying some fraction of the deaths from that cause to occur at an older age. This assumes that a predicted decline in mortality for certain diseases due to medical advancements or lifestyle changes is likely to reduce or postpone death by those diseases.
The effects of delayed mortality by cause can be estimated by assuming that mortality rates by cause are shifted to those observed for younger ages. For example, a five-year delay in mortality by heart disease could be estimated by shifting from the observed mortality rates by heart disease to those for an age five years younger. So, for 65 year olds, the relevant mortality rates would be the pertinent estimates for 60 year olds.

Note that cause-delay models are similar in philosophy to projection by reference to a more “advanced” population, except that instead of targeting the mortality of a different population, cause-delay models target a different sector of the mortality for the same population.

7.2.6 Epidemiological

An epidemiological model analyzes the relationship between specific risk factors and their effects on mortality. Some examples of appropriate risk factors include smoking, obesity, socio-economic status and specific diseases. The approach is similar to cause-delay but here the focus is on the impact of specified risk factors rather than causes of death.

After estimating the impact of each risk factor on mortality rates, mortality forecasts can be produced by projecting these risk factors into the future, assuming distributions which may change over time. Although there have been recent developments in modeling demographic and epidemiological processes of mortality, more accurate estimation of the relationship between risk factors and mortality may be required for these models to be widely adopted (Andreev & Vaupel, 2006).

A number of studies have been conducted comparing various mortality forecasting methodologies. For instance, Wong-Fupuy and Haberman (2004) have examined a variety of deterministic and stochastic mortality models in a theoretical sense. However, as far as we are aware, there have been no quantitative model comparisons. In conjunction with the UK academics and mortality experts Andrew Cairns, David Blake and Kevin Dowd, JPMorgan undertook a quantitative comparison of various stochastic mortality models. The resulting working paper is available for download from the LifeMetrics website.

Eight models were evaluated in the study, representing a range of both new and existing models. They can be classified as follows:

1. Lee-Carter model (1992)
2. Renshaw-Haberman model (2006), a generalization of Lee-Carter to include a cohort effect
4. P-splines model (Currie et al., 2004)
6. A number of extensions to Cairns, Blake and Dowd (Cairns et al., 2007)

A brief description of these models follows. The reader can refer to Cairns et al. (2007) for a detailed discussion of all of the models. As discussed in the paper, some of these models provide better fit to certain data sets. However, the choice of model should not be driven by quality of fit alone. Consideration should be given to the purpose of the forecast and the dynamics of mortality. Other factors to consider include robustness and qualitative criteria such as transparency and parsimony.

As part of the LifeMetrics toolkit, software is available to aid users in calibrating and forecasting mortality using the listed models (except for the P-spline model, an implementation of which is already available free of charge from the UK CMIB). We provide this modeling software to assist users in their understanding of historical mortality and in forecasting future mortality. The user is reminded that a forecasting model should be selected carefully and calibrated thoughtfully to ensure it is both appropriate and meaningful.
7.3.1 Stochastic mortality models

In this section, we describe the eight models analyzed in Cairns et al. (2007). In these models, the following notation is used:

- \( \beta_x^{(i)} \) functions reflect age-related effects, where \( x \) denotes age
- \( \kappa_t^{(i)} \) functions reflect period-related effects, where \( t \) denotes period or time
- \( \gamma_c^{(i)} \) functions reflect cohort-related effects, where \( c = t - x \) denotes birth cohort

7.3.1.1 Lee-Carter

In 1992, Ronald Lee and Lawrence Carter proposed an extrapolative model for use in forecasting future mortality where the model parameters were calibrated to historical mortality experience (Lee & Carter, 1992). In summary, it is a one-factor model of the mortality surface (age and time), where the general level of mortality is assumed to be a stochastic process in time, and where the age distribution of mortality is assumed to be deterministic and is calibrated from historical data. By "one-factor", we mean that there is a single dynamic process which drives the changes in mortality rates for all ages from one year to another.

The Lee-Carter model for mortality can be expressed in the form:

\[
\log(t, x) = \beta_x^{(1)} + \beta_x^{(2)} \kappa_t^{(2)} + \gamma_{c-x}^{(3)}
\]

where the \( \beta_x^{(1)} \) coefficients describe the age-specific pattern of mortality (i.e., the current mortality table), and the \( \beta_x^{(2)} \) coefficients describe the sensitivity of mortality rate by age to changes through time, specified by \( \kappa_t^{(2)} \).

In terms of stochastic forecasting models, Lee-Carter (LC) is the simplest implementation of a range of models that incorporate varying degrees of complexity in order to model perceived age, period and cohort effects in the data. The disadvantage of more complicated models is that they can require careful calibration and the larger number of parameters can make it more difficult to gain an intuitive sense of how the model reacts to changes in parameters.

7.3.1.2 Renshaw-Haberman

The Renshaw-Haberman model is a generalized version of the Lee-Carter model which includes a cohort effect (Renshaw & Haberman, 2006). It can be expressed in the form:

\[
\log(t, x) = \beta_x^{(1)} + \beta_x^{(2)} \kappa_t^{(2)} + \beta_x^{(3)} \gamma_{c-x}^{(3)}
\]

where the addition of the final term relative to LC is used to model a cohort effect in terms of a dynamic which is dependent on year of birth, \( \gamma_{c-x}^{(3)} \). The impact of this cohort effect can be varied by age through \( \beta_x^{(3)} \).

7.3.1.3 Currie Age-Period-Cohort Model

Currie (2006) proposes what amounts to a simplified version of the Renshaw-Haberman model, where the age, period and cohort effects influence mortality rates independently. The model can be expressed in the form:

\[
\log(t, x) = \beta_x^{(1)} + \kappa_t^{(2)} + \gamma_{c-x}^{(3)}
\]

7.3.1.4 P-Splines

In the UK, the P-Spline model has been particularly high profile in the last year due to the release of a free software implementation by the CMIB. The P-Spline approach is essentially a penalized fitting process using basis splines (Currie et al., 2004).

Basis splines are a set of basis functions constructed from cubic splines. They are fitted to the underlying data through the optimization of a penalized likelihood or regression function and have the form:

\[
\log(t, x) = \sum_{i,j} \theta_{ij} B_{ij}^y(x, t)
\]

where \( B_{ij}^y(x, t) \) are B-spline basis functions and \( \theta_{ij} \) are weights associated with each function. The fitted set of splines can be extrapolated forward in time enabling forecasting. By measuring volatility in the estimation process, confidence levels around the central mortality estimate can also be produced. It is important to note, however, that the methodology does not enable direct stochastic simulation of future mortality rates, although a common misperception is that this is possible. Simulation of the actual number of
deaths (based on the Poisson process applied to the central mortality estimate) is of course possible.

7.3.1.5 Cairns, Blake & Dowd

Cairns, Blake and Dowd fit a model direct to initial mortality rates instead of central mortality rates (Cairns et al., 2006). The simplest version, of the model, has the form

\[ \logit q(t,x) = \kappa_1^t \kappa_2^x (x - \overline{x}) \]

where \( \overline{x} \) is the mean age over the range of ages used in the analysis. There are a number of extended versions of the model, with varying additional terms to cover a cohort effect with a linear, quadratic or time-dependent variability. These models differ from the previous stochastic models (Lee-Carter, Renshaw-Haberman, Currie) in that they assume a functional relationship between mortality rates across ages. See Cairns et al. (2007) for more details on this class of models.

7.4 Practical applications of mortality models

As discussed in Chapter 4, national and international bodies make periodic population projections. These involve estimation of future mortality rates and, here, we review their methodologies in these terms. In addition, we review the practice of the actuarial profession in both the UK and the US.

7.4.1 United States - Social Security Administration

In the United States, the Social Security Administration (SSA) is the governmental organization which provides social retirement benefits through its Old-Age, Survivors, and Disability Insurance (OASDI) programs. Each year, the SSA produces a report on the financial health of the OASDI programs (Bell & Miller, 2005). Integral to this report is a projection of mortality for Americans participating in the programs.

The model for projecting mortality in the SSA’s reports is an extrapolative model which includes expert opinion. The rates are based on cause-specific death rates. Specifically, the trend in mortality improvements by age, sex, and cause of death are first calculated for the past 20 years. These reductions in mortality are assumed to continue for a short period, such as two years. After that period, the reductions in mortality are projected to trend down to some predetermined ultimate improvement rate, which applies in, say, 30 years time. By applying these projected improvements to the current mortality rates, future mortality rates and hence life expectancy can be estimated. Therefore, these projections assume that the decline in mortality will not continue at its current pace, but will slow and then level off.

Note that the SSA projects three separate scenarios of future mortality improvement to take into account the potential variability of future forecasts: an expected case, a high mortality case, and a low mortality case. As discussed in the 2005 Annual Report from the SSA’s Board of Trustees (2005), the ultimate mortality improvement rates are 0.71% in the expected case, 0.33% for the high case and 1.23% for the low case.

7.4.2 United Kingdom - Office for National Statistics

Biannually, the Office for National Statistics (formerly the Government Actuary’s Department, GAD) produces population projections for the UK (GAD, 2006), and estimates of future mortality are integral to this projection.

In recent forecasts, the GAD has used a model very similar to the one used by the SSA, although not based on specific cause of death. The GAD’s method incorporates trend extrapolation with a target future mortality improvement rate. Note that like the SSA, the GAD targets rates of mortality improvement rather than mortality rates themselves.

The GAD determines the current level of mortality and then projects future rates of mortality improvements. These improvements are initially set to the recent average of mortality improvements, and then trend down to a fixed level of 1% per annum from 2029 onwards. This trend is assumed to be more rapid than a linear trend for males and less rapid than a linear trend for females. Projections are undertaken by cohort for those born before 1960 and by calendar year (i.e., period) for the remainder.

The GAD projects high and low variant scenarios. In these, the final fixed improvement rates are set to 2% and 0% instead of the 1% assumed in the base scenario. In addition,
the initial improvements are assumed to be 2% higher and 2% lower than the base level respectively.

7.4.3 Global - World Health Organization

The World Health Organization recently funded a report on the projection of global mortality (Mathers & Loncar, 2006). This report estimated future mortality from 2002 to 2030 for approximately 100 countries worldwide. Unlike the SSA and GAD models, which were basically extrapolative in nature, the models used by the WHO researchers were based on epidemiological structures. For each of a small number of major cause of death groupings, mortality rates were assumed to be related to a number of socio-economic and other health factors.

Specifically, there were four factors incorporated into the model: average income per capita, average number of years of education, time (as a proxy for scientific and technological advancement) and tobacco consumption. These factors were incorporated into the model using the historically observed relationships between them and cause-specific mortality rates. Note that the model assumes that trends in future mortality for poorer countries will be related to economic and social development in a similar manner to that experienced in more wealthy countries.

Like the SSA and GAD projections, the WHO study also projected three scenarios: baseline, optimistic, and pessimistic.

7.4.4 Projections prepared by the actuarial profession

7.4.4.1 United Kingdom - Methods developed by the CMIB

Since the publication of the a(55) tables in 1953, the CMIB has customarily provided mortality projections for annuitants and pensioners when it has published mortality tables. For the “80” and “92” Series (two sets of mortality tables based on the 1979-82 and 1991-94 experiences respectively) an extrapolative projection method was used to introduce an explicit formula for the reduction factor to allow for mortality projection for annuitants and pensioners. The method adopted by the CMIB to estimate future rates of mortality at time t was to multiply the mortality rate from the base table \( q_{x,t} \) by the corresponding reduction factor \( RF(x,t) \). The reduction factor being expressed as follows:

\[
RF(x,t) = \alpha(x) + [1 - \alpha(x)][1 - f(x)]^{t/\gamma}
\]

The most recent set of projections (those proposed to be used with the ’00’ series) were distinctive in three important aspects as compared to the corresponding sets for the “80” and “92” Series:

- They recognized the cohort effect; defined as the dependence of mortality improvement rates on a person’s year of birth
- Three alternative projections were offered, instead of the traditional single projection, based on alternative assumptions on the longevity of the cohort effect
- Base projections were not directly quoted but software for their analysis was provided

7.4.4.2 United States - Methods developed by the Society of Actuaries

The Society of Actuaries recommended a projection basis to be used for statutory reserving for the first time with the GAR94 tables for annuitants (Fupuy and Haberman, 2004).

The Society of Actuaries adopted an extrapolative approach similar to that used by the CMIB for its “80” and “90” series. The 1994 mortality tables were published as a base table with associated age-dependent mortality improvement factors (Scale AA\(x\)). The projected probabilities of death were calculated as follows:

\[
q_{x,y} = q_{x,1994}(1 - AA_x)^{y-1994}
\]

For more details on the technical aspects involved in the calculations of the improvement factors, the reader can refer to the Society of Actuaries Group Annuity Valuation Table Task Force (1995).

7.5 Model selection

As we have seen earlier in this chapter, there are many approaches and models that have been proposed to project
mortality rates. Selecting the method to be used should depend on the data and their reliability, the resources available for the project and the purpose for which the projection is required. In general, no mortality projection basis can ever be considered “correct”. It is worth noting that many of the projection methods discussed in this chapter suffer from potential drawbacks. For example, and as noted by Brouhns et al (2002a), univariate extrapolation of the parameters of a mortality model can be misleading, and while a multivariate time series model for the parameters is possible, it can lead to computational intractability.

While the methodology suggested by Lee & Carter (1992) avoids these problems, it implicitly assumes - by using the ordinary least square to estimate the parameters - that the errors are homoskedastic, which is unrealistic in that the logarithm of the observed mortality rate is much more variable at older ages.

Also, we have to bear in mind that regardless of the method of projection used, when projecting mortality at very old ages several problems arise, in particular, because of inaccuracies in the data available and variability due to small exposures to risk.

One important consideration in model selection is whether the model should capture long-term trends or “tail-event” shocks. For example, if the forecaster is concerned with the impact of a possible pandemic, a model specifically designed to incorporate catastrophic “shock” mortality would be appropriate. As there is limited historical data on shocks, expert opinion should generally be incorporated (Van Broekhoven, 2002).

Additionally, modelers should review the qualitative trade-off between simplicity and accuracy. Cairns et al. (2007) provide a list of qualitative criteria that should be considered in the model selection process. Finally, note that because of the significant uncertainty surrounding mortality forecasts, models should include “high-medium-low” scenarios, or produce stochastic forecasts. The ability to easily produce such scenarios may be a further factor in model selection.
8. Quantifying exposures to longevity and mortality

8.1 Introduction

This chapter addresses two important steps in the LifeMetrics Framework that we have not yet covered, namely, Step 1 (Determine the nature of the exposure) and Step 4 (Quantify risk).

We begin in the next section by describing a few of the common types of exposure to longevity and mortality, including defined benefit (DB) pension plans, annuity portfolios, life insurance books and mortality-linked investments. We then present a detailed discussion of how to use the LifeMetrics Index and other data sources to evaluate the sensitivities to longevity and mortality, and quantify the risks associated with these exposures. The examples used throughout the chapter to illustrate the risk calculations are based on DB pension plans and annuity portfolios, but clearly the framework can be applied to other exposures in a completely analogous way.

Informed readers may wish to skip the descriptions in the next section and launch straight into the material on quantifying the impact of mortality and longevity risk on the exposure.

8.2 Characterizing the nature of the exposure

8.2.1 Long vs. short exposure

The first step in managing any risk is to understand the nature of the exposure to that risk.

In describing the exposure to a particular risk, the financial markets use the terms “long” and “short”. A “long” exposure means that one will benefit from an increase in the underlying variable and a “short” exposure means that one will benefit from a fall in the underlying variable. For example, an equity investor who is long equities will benefit from an increase in equity prices, whereas an investor who is short equities will lose money from an increase in equity prices.

According to this terminology, DB pension plans (or more accurately, their sponsors) are naturally “short longevity”, meaning that they benefit if longevity falls and lose if longevity increases. Alternatively, one could also describe the DB pension plan as being “long mortality”, since they benefit if the mortality rate increases. Similarly, the annuity business of an insurer has the same short exposure to longevity as a DB pension plan.

By contrast, a life insurance portfolio (involving, for example, term life and whole life policies) has the opposite exposure and is “long longevity”, or equivalently, it is “short mortality”. Such a life portfolio will benefit if people live longer than expected (lower mortality) and experience a loss if people die sooner than expected (higher mortality).

So we can conclude that the life business of an insurer has, broadly speaking, the opposite longevity exposure to a pension plan or an annuity business. In other words a life business provides a kind of “natural hedge” - albeit an imperfect one - of a pension or an annuity business. While this has been familiar to insurers and academics for some time, there have been few systematic attempts to measure the effectiveness of this natural hedge and quantify the residual risk. As we show in a case study in Chapter 9, the LifeMetrics framework enables the offsetting nature of these two types of exposures to be evaluated and the effectiveness of the natural hedge to be quantified.

8.2.2 Defined benefit pension plans

A pension plan is an arrangement typically set up by a sponsor (an employer, government or industry association) that pays benefits to employees after their retirement. In a defined benefit pension plan, the level of benefit paid to an employee post-retirement is prescribed by the plan and paid for as long as the employee lives. The sponsor bears the risk that sufficient funds will continue to be available in the future to pay the pensions promised to their current and former employees.

A DB pension plan is similar to a portfolio of annuities (see below) paying out fixed, or inflation-indexed, amounts to a population of individuals from retirement for the rest of their lives. Although at first glance the longevity risk associated with a DB pension plan is the same as that for a simple
annuity portfolio, it is in fact more complicated than that. A typical DB pension plan also has other longevity and mortality-linked features that alter the risk profile from that of a vanilla annuity portfolio. In particular, most DB plans are exposed to additional longevity and mortality risks through the following features:

- Spouse’s pension
- Dependents’ benefit
- Death benefit

The spouse’s pension is typically a percentage of the beneficiary’s pension (e.g., 50%) that is paid to the spouse (if there is one) on the death of the beneficiary in retirement. A similar benefit may also be paid for dependent children. Furthermore, before retirement the beneficiary may also be entitled to a death benefit. These additional dependencies on longevity and mortality complicate the overall exposure of the pension plan to longevity and mortality risk and need to be taken into account.

The population of members of a pension plan are often classified into three groupings:

- “Pensioners” or “retirees”: former employees who have retired and are drawing a pension
- “Actives”: employees who are still working and accruing benefits
- “Deferreds” or “deferred vesteds”: employees who have accrued pension benefits in the past but have not yet retired

The analysis of pension liabilities and longevity risk is often performed separately on these groups. Apart from the benefits for spouses, dependents and death in service, the longevity risk associated with pensioners is identical to that associated with immediate annuities (see below), whereas that associated with deferreds is identical to that of a deferred annuity (see below).

In order to meet the future obligations to pensioners, pension plans maintain a portfolio of assets which they invest to generate returns. So overall they are exposed to the combined impact of risks on the assets and risk on the liabilities. The risks on the assets are investment risks, which typically include equity risk, interest rate risk and credit risk, perhaps with some exposure to the risk associated with alternative assets such as real estate, hedge funds and private equity. The risks on liabilities include longevity and other demographic risks (such as early retirement and lump sum election), as well as interest rate risk and —if inflation linked— the risk associated with both realized inflation and inflation expectations. Simulation of the total risk facing a DB pension plan must incorporate all these risk factors.

### 8.2.3 Annuity contracts

In an insurance context, the term “annuity” is used to describe a periodic payment made to an individual - the “annuitant” - for as long as that individual lives. This is typically in return for a single up-front premium (sometimes called single-premium annuities). For example, an annuitant might pay $100 at the start of the annuity and then receive $7 per year until he or she dies. An annuity has, therefore, the same exposure to longevity as a basic defined benefit pension plan.

It is useful to distinguish between what are called “immediate annuities” and “deferred annuities”. Immediate annuities are annuities that start at the time of purchase; the premium is paid and at the end of the first period (usually the end of the month) the first payment is made to the annuitant. By contrast a deferred annuity is an annuity that starts at a future date, e.g., at an individual’s retirement date. Clearly however, there is no difference between these two types of annuities once they have actually started and are “in payment”.

There are two key risks that impact both types of annuities: (i) longevity risk and (ii) interest rate risk. Longevity risk directly impacts the cash flows that must be paid out by the annuity provider, since increasing longevity (due to falling mortality rates) means the annuity payments must be made for longer than expected. This clearly has a significant effect on the value of the annuity. By contrast, interest rate risk impacts on the value of the annuity and not the cash flows.

In situations where interest rates are stable and at relatively low levels, such as in Western Europe, North America, Japan, etc., longevity is the dominant risk for immediate annuities. This is because the combination of low interest
rate volatility and the relatively short interest-rate duration of immediate annuities leads to a relatively modest impact on the volatility of the annuity value. However, for deferred annuities it is interest rate risk that is generally the dominant risk, because of the much longer duration of these contracts (Richards and Jones, 2004).

The underlying mortality risk exposure for an insurance company’s annuity portfolio is given by the formulae below which reflect the value of annuities, based on a cohort mortality table.

Annuities in payment paying $1 per year:

\[ a_{x,t} = \sum_{i=t}^{\infty} DF_{t,t+i} \cdot \prod_{j=0}^{i-1} (1 - q_{x+j,t+i}) \]

Deferred annuities starting in \( n \) years time paying $1:

\[ n \cdot a_{x,t} = \sum_{i=n+1}^{\infty} DF_{t,t+i} \cdot \prod_{j=0}^{i-1} (1 - q_{x+j,t+i}) \]

In these equations \( DF_{t,t+i} \) denotes the discount factor from time \( t+i \) back to time \( t \), which is used to present value each annuity payment.

### 8.2.4 Term life exposure

Term insurance is the simplest traditional life insurance product. It provides a death benefit only for a limited period of time, known as the “term” of the policy. For example, 20-year term insurance provides, in return for a premium, a lump-sum death benefit if the insured life dies during the 20-year term of the policy.

Term insurance comes in different varieties, such as:

- Single premium, which is paid once at the start of the policy
- Regular premium, which is paid monthly or annually until death or maturity
- Level term, where the premium payments and death benefit are fixed
- Decreasing term, where the premium payments are fixed, but the death benefit decreases through time

The value of a single-premium term insurance contract paying $1 death benefit with a term of \( n \) years is:

\[ A^1_{x,t:0} = DF_{t,n+1} \cdot q_{x,n+1} + \sum_{j=1}^{n} DF_{t,n+j} \cdot q_{x+j,n+j} \prod_{j=0}^{n-j} (1 - q_{x+j,n+j}) \]

### 8.2.5 Investments in longevity and mortality

There have only been a limited number of structures available to capital markets investors interested in taking exposure to mortality and longevity risk. Most public issues have been in the form of short-term mortality risk transfer vehicles, such as mortality catastrophe bonds, which are intended to transfer to investors the risk of a short-term rise in mortality rates (due to, say, an influenza pandemic). Despite being linked with an esoteric risk typically unfamiliar to investors, these mortality “cat” bonds have been well received, primarily due to a low probability of loss coupled with attractive returns, investment diversification and conceptually intuitive structures.

The cat bonds issued to date have been structured as principal-at-risk notes with a fixed tenor, where the principal repayment is contingent on a catastrophic outcome for the value of a customized mortality index. The catastrophic event is defined as an extreme increase in mortality beyond a particular baseline mortality rate. Cat bonds have been issued primarily by insurance companies looking to free up capital related to the extreme mortality risk they face in their life insurance books. Table 8.4 at the end of this chapter lists the public cat bond issues that have been brought to market over the past four years.

In contrast to mortality cat bonds, efforts to transfer longevity risk to investors have been more limited and, so far, unsuccessful. Longevity bonds, or survivor bonds, were first suggested by Blake and Burrows (2001), who proposed an annuity structure where annual payments were tied to the survivorship index of a reference population. Since then, there has only been a single public attempt to issue a longevity bond and that was ultimately abandoned. In November 2004 the European Investment Bank (EIB) unveiled plans to issue a 25-year bond linked to an index based on the longevity of a cohort of England & Wales males aged 65 in 2003. The bond, structured by BNP Paribas, was targeted at annuity providers and pension plans to provide a hedge of their longevity risk. It was unsuccessful for a number of reasons including (i) the structure of the bond (one cohort
of 65-year-old males made for a poor hedge of longevity for an annuity book or pension plan, unlevered exposure to longevity risk meant that it required a large amount of up-front capital for the level of protection it offered, no final settlement at maturity to reflect longevity risk in the liabilities beyond 25 years), (ii) the receptivity of the investor community (novelty of the idea, limited recognition of the threat posed by longevity risk), and (iii) a low yield.

An existing arena for investments in longevity is the life settlements market and its related securitizations. Life settlements are transactions where individuals transfer life insurance policies that they do not want or need to third party investors instead of surrendering them back to the insurance company. This can be beneficial to both investors and policyholders when the surrender value of a life insurance policy is lower than its economic value. Policyholders, therefore, receive more than the surrender value of the policy and investors obtain exposure to longevity risk at a competitive price. Investors earn a return unless the insured individuals live longer than the life expectancy implied by the purchase price of the structure. Recently, insurance brokers and financial service providers have partnered to pool life settlements and package them for securitizations. While the returns on life settlement-backed securitizations can be attractive, there are several risks that make potential investors wary. In particular, the structures are complex, not transparent, dependent on the idiosyncrasies of each individual policy, and require extensive legal and actuarial work. They are also based on a relatively small number of lives which means there is considerable idiosyncratic sampling risk (see later) associated with the longevity exposure.

The life settlements market shows that longevity risk is an attractive risk class for sophisticated investors seeking to earn returns uncorrelated to other asset classes. It is likely to only be a matter of time before pension and annuity sourced longevity exposure also becomes available to investors. But regardless of its source, investors, like other market participants, need a common platform with which to assess risk and return of longevity-linked and mortality-linked investments.

8.3 Mortality data for evaluating exposures

8.3.1 Base mortality vs. forecast mortality

The valuation of a life-contingent exposure, such as the liabilities of a DB pension plan, annuity portfolio, or life insurance book, involves making two kinds of assessments concerning longevity and mortality levels:

- Current, or base, mortality rates
- Future, or forecast, mortality rates

By “base mortality rates” we mean the current period life table that is appropriate for the population in question. In the language of Chapter 6, they correspond to the current mortality curve, based on graduated initial mortality rates. By contrast, the forecast mortality rates reflect what is called a “best estimate” for the future evolution of mortality rates, typically involving improvements over time relative to the current period life table. While the base mortality can be determined from experience data for the population (if they are available), the forecasted mortality is highly subjective and dependent on individual viewpoints. As emphasized in the previous chapter, forecasts of future mortality can vary significantly.

A recent comparative study by Cass Business School of mortality assumptions used in the estimation of corporate pension liabilities across the European Union, the US and Canada showed significant variations across countries (Verall, Sithole and Haberman, 2006). This variation is reflected in a comparison of life expectancy for 65-year-old males calculated from these assumptions, which ranged from 24.2 years for France down to 15.1 years for Denmark. For the UK the figure was 21.0 years and for the US 17.6 years. This variation in life expectancy results is, unfortunately, unlikely to have been the result of careful deliberation and analysis based on the specific characteristics of each country’s pension population. Rather, it is likely to be due in large part to corporations in some countries using outdated life tables and/or underestimating improvements, while those in other countries are using more up-to-date tables. In the absence of clear accounting or regulatory guidance, many pension plans have taken a mortality table off the shelf, and applied it without sufficient analysis of how mortality im-
provements are likely to improve in the future for their specific population.

By contrast, a combination of insurance regulation, best practice defined by industry bodies and the profit motive have ensured that insurance companies, unlike pension plans, have been generally more consistent and up-to-date in their use of mortality tables and improvement factors.

8.3.2 Valuation methodology

The net result of combining both base and forecast mortality is a graduated cohort life table that can be used to produce a best estimate for the pension plan’s liability cash flows out into the future. When confronted with best estimate cash flows it is important for the user to bear in mind that these have been based on a particular view about future mortality rates that may be overly optimistic or overly pessimistic. In fact, the valuations of most defined benefit pension plans in the Western World traditionally have been based on out-of-date and thus overly optimistic assumptions about future mortality improvements. This means that the majority of liability valuations for these pension plans significantly underestimate the true size of the liability in economic terms.

Valuations of exposures performed by actuaries generally involve using graduated “best estimate” mortality rates to determine the expected future cash flows, which are then discounted back to the present at a particular discount rate. For pension plans the discount rate used in reporting accounting valuations is typically a AA-rated long-term corporate bond yield. For an annuity portfolio the discount rate used by insurance companies is typically much lower, implying a liability value much larger than for a pension plan with the same cash flows. These valuations are different partly because insurers incorporate in the discount rate: expenses, profit margin and a risk premium. This risk premium is compensation for the insurer bearing all the risks associated with the portfolio, including longevity and other demographic risks, as well as reinvestment risk on the assets.

8.4 Evaluating the impact of longevity and mortality risk

8.4.1 Overview

An essential part of risk management is evaluating the impact of key risk factors on the exposures. In particular, for a DB pension plan this means evaluating the sensitivities of the liability cash flows and their value to changes in longevity. Similarly, for a life insurance book this means evaluating the sensitivity in cash flow and value to changes in mortality. The starting point for this is a set of forecasts for the evolution of future mortality rates which are derived from both of the following:

• A simulation of future paths for mortality using a stochastic mortality model, and
• A set of stress tests in which the best estimate path of mortality is shocked in an adverse way

These paths are then applied to the exposures so that the impact on cash flow and value can be determined, along with the risk to the exposures in terms of both cash flow and value.

8.4.2 Demographic risks vs. financial risks

There are many different types of risk and many different ways of classifying them. One simple classification that is pertinent to the current situation is the distinction between financial and demographic risks.

8.4.2.1 Financial risks

Financial risks are based on the movements in prices of assets that are traded in the financial markets, including interest rate risk, inflation risk, credit risk, equity risk and other investment risks. These risks are generally transparent and have a wealth of historical data behind them, (although there are exceptions to this, for example, hedge fund investments).

8.4.2.2 Demographic risks

Demographic risks are risks associated with an underlying population that have until recently only been traded in the
insurance markets. Longevity risk and mortality risk are examples of demographic risks. Others include early retirement risk (retiring before the normal retirement date) and lump sum election risk (taking a lump sum instead of an annuity at retirement), both of which may be faced by DB pension plans. Another example relates to insurance portfolios which face the demographic risk that policyholders “surrender” their policies early before they mature. Mortgage portfolios and mortgage-backed securities (MBS) also face demographic risks in the form of prepayment risk.

### 8.4.2.3 Population basis risk

A particular type of demographic risk facing pension plans and life-contingent insurance products is population basis risk. This is the risk associated with the difference in longevity/mortality experience between two populations. For example, populations with differing profiles of age, gender, socioeconomic group, lifestyle and geography will have differing profiles of longevity and mortality risk. All pension and life insurance populations have basis risk relative to the national population and also relative to whatever standard mortality table that they use for valuation. The longevity and mortality risk for these specific populations can be decomposed into two components:

- The risk associated with the national population, plus
- The basis risk of the specific population relative to the national population

Population basis risk can also be a concern in hedging and risk transfer transactions. If the population associated with the hedge has a sufficiently different profile from that associated with the underlying exposure, then this basis risk will lead to a lower degree of hedge effectiveness than would otherwise be the case. Note that basis risk does not necessarily mean the hedge is ineffective, but in designing such hedging solutions it is important to ensure any basis risk is acceptably low. This is discussed in more detail in the next chapter in the context of hedge effectiveness.

### 8.4.3 The nature of longevity risk and mortality risk

The risk associated with future mortality rates falls into four distinct categories:

- Volatility
- Sampling risk
- Jump risk
- Trend risk

#### 8.4.3.1 Mortality volatility

The first of these, volatility, refers to the year-on-year fluctuations in mortality rates that come from the underlying process of mortality and noise in data collection. This category of mortality risk is generally quite stable through time.

The charts in Chapter 6 show just how volatile mortality rates can be from year to year. Table 8.1 lists the volatility of mortality rates for selected sub-populations, where we have used the usual definition of volatility as the standard deviation of percentage changes in the crude central mortality rate. The volatility of mortality for 65-year-old males is 2.84% in England & Wales (1961-2005) and 1.99% in the US (1968-2004). For 80-year-old males the volatilities are slightly higher: 4.04% and 3.58% respectively. Note that the volatility falls if the age group is broadened to include a range of ages on either side, as some of the noise associated with mortality at individual ages is eliminated. This “bucketing” of ages has a similar effect to graduation of mortality rates across age.

Mortality improvements are not independent through time. They typically show a strong negative autocorrelation from year to year, which suggests that high improvements are generally followed by low improvements and vice versa.

The impact of volatility of mortality rates on exposures is lower than might be expected from crude mortality data
because of one or more of the following reasons, all of which reduce the noise inherent in exposure valuations:

- Smoothing of mortality rates across age in producing graduated life tables (as described in Chapter 5 and Appendix A)
- Smoothing of mortality rates over time in producing graduated life tables (e.g., using a three year period rather than a one-year period)
- The fact that most exposures reflect not just one age but a range of different ages (e.g., 60-69 year olds).

8.4.3.2 Mortality sampling risk

Sampling risk reflects the statistical uncertainty associated with mortality rates for small populations. For a large population (more than 100,000 lives) the realized mortality experience each year is generally very close to the true mortality rate. This means two things:

- We can be confident that — barring a very rare mortality shock — any estimate of mortality based on actual experience is likely to be accurate (estimation error is low)
- Next year’s mortality experience — barring a very rare mortality shock — is likely to be very close to the expectation under the true mortality rates (there should not be any large short-term surprises)

For small populations (e.g., less than 1000 members) the opposite is true. For example, in the extreme case of when there is only one person in the sub-population then the realized mortality experience in any year will be either 0% or 100%, regardless of what the true underlying mortality rate actually is. So for small populations it is important to note that:

- Any estimate of mortality based on actual experience has a large estimation error associated with it
- Next year’s experience can be far from expectation, even if the true mortality rate is known with certainty

8.4.3.3 Mortality jump risk

In contrast to volatility, mortality jump risk refers to sharp movements in mortality rates from one year to the next, such as that exemplified by the 1918 influenza pandemic. Jump risk is typically thought of as a one-sided risk with actual mortality rates more likely to jump to higher levels (due to a pandemic, natural disaster or terrorism), rather than to lower levels. This is because causes of mortality improvements (e.g., cures for diseases, healthier lifestyles, etc.) generally evolve gradually and even when fully developed, their impact on individual mortality is not instantaneous. Hence, a sharp downward jump in realized mortality rates is much more unlikely than a sharp upward jump. (Note that a sharp downward jump is, however, likely to follow an upward jump as mortality rates return to their long-term trend).

In the context of jump risk it is important to distinguish between changes in realized mortality and changes in expectations about future mortality. The announcement of a cure for cancer may prompt a sharp downward jump in expected future mortality, leading to a sharp rise in the value of pension liabilities and a sharp fall in the value of life insurance liabilities. However, realized mortality is likely to improve only slowly as the cure will take time to roll out and take effect.

8.4.3.4 Mortality trend risk

Mortality trend risk is a long-term risk that reflects the uncertainty in the trend of future mortality rates. This is a two-sided risk in that long-term mortality improvements may be higher or lower than the historical trend and higher or lower than forecasts.

The impact of trend risk on realized mortality rates over the short term is small, but its impact on expectations about future mortality can be very large. So, whereas realized mortality may change very slowly, the value of liabilities may move very sharply if expectations about the trend of mortality improvements changes.

8.4.4 Simulation of mortality risk

Stochastic models for simulating mortality risk vary depending on the nature of the mortality exposure and the time horizon over which the risk is being measured. For example, for simulations over short time horizons it is generally not necessary to model trend risk in a sophisticated manner, as its short-term impact is small. In a similar fashion, when simulating mortality for large populations it is not
necessary to model the sampling risk, but this must be incorporated in simulations for small populations.

8.4.4.1 Model risk and parameter risk

When simulating mortality, another risk arises that we have not discussed above, namely, model risk - the risk that we have incorrectly chosen the model.

A related risk is parameter estimation risk. Parameter estimation risk refers to the uncertainty associated with the calibration of the model, in which the model’s parameters are fitted to appropriate data. Note that, for long-term simulation models, trend risk, in addition to reflecting the uncertainty in the true long-term direction of mortality improvements, also has an aspect that is essentially parameter estimation risk (in terms of errors in the calibration of the model to the historical trend).

8.4.4.2 Mortality catastrophe risk vs longevity risk

The nature of the simulation may also change depending on the exposure. For example, the risk associated with mortality catastrophe (or “cat”) bonds (e.g., Swiss Re’s Vita bonds) requires very different simulation models from the longevity risk associated with a pension plan or annuity. Mortality cat bonds are relatively short-term investments that are subject to losses if mortality rates jump significantly higher than current levels. For example, the original Vita bond issued in 2003 was a three-year investment in which investors started suffering losses if a customized mortality index exceeded 130% of the 2002 baseline level. To capture this jump risk requires a so-called “jump-diffusion” model of mortality rates which captures the probability of large jumps in mortality due to pandemics, natural disasters, etc. along with the mortality volatility (see, for example, Lin and Cox, 2006).

By contrast simulation models for the longevity risk of pension plans and annuity books usually do not need to incorporate jump risk, as it has limited impact on the downside value of the liabilities. However, simulation models for these exposures must incorporate trend risk, as well as the parameter estimation risk associated with other parameters in the model.

8.4.4.3 Historical simulation of mortality

A particularly simple simulation method that can be applied to mortality rates is known as ‘historical simulation’. Historical simulation involves first calculating the relative changes in historical mortality rates and then applying those changes to current base mortality levels to generate future scenarios (see, for example, Mina and Xiao 2001). It is widely used by financial institutions in risk simulations of FX rates, interest rates, equity prices, etc. It can be applied in two different ways (Coughlan, Kolb and Emery, 2003):

- “Longitudinal historical simulation”, which produces a single path for longevity over a long time horizon
- “Cross-sectional historical simulation”, which produces many scenarios for longevity over just a single period (the next year)

As an example, suppose we have 41 annual historical mortality rates together with the current rate: \( q_{1965}, q_{1966}, \ldots, q_{2005} \). Here the subscript denotes not age but year (period). The first step in historical simulation is to calculate the relative changes (or ‘returns’) in historical rates, which can be done in a number of different ways. A common way of calculating these relative changes is to take the natural logarithm of the ratio between consecutive mortality rates as follows:

\[
rt = \log \left( \frac{q_{1965+t}}{q_{1965+t-1}} \right), \text{ where } t = 1,2,3,\ldots,40
\]

The next step is to apply these changes to the current mortality rate. The result is a series of forward-looking scenarios for the mortality rate, denoted \( z_t \).

\[
\begin{align*}
Z_{2006} &= \exp (r_1 \cdot q_{2005}) \\
Z_{2007} &= \exp (r_2 \cdot Z_{2006}) \\
Z_{2008} &= \exp (r_3 \cdot Z_{2007}) \\
&\text{etc.}
\end{align*}
\]

This procedure generates a single scenario path for this mortality rate at each future time point over the next 40 years: \( Z_{2006}, Z_{2007}, \ldots, Z_{2045} \). Because the scenarios define a single path for future rates we call this longitudinal historical simulation (see Figure 8.1).

There is another way of generating scenarios from the relative changes in historical rates, which produces multiple
Figure 8.1 The principle of longitudinal historical simulation

Figure 8.2 The principle of cross-sectional historical simulation

Figure 8.3 Longitudinal historical simulation applied to EW mortality rates for 65-year-old males

Figure 8.4 Histogram of mortality rates in one year’s time for EW 65-year-old males based on cross-sectional historical simulation

Figure 8.5 Longitudinal historical simulation applied to US mortality rates for 65-year-old males

Figure 8.6 Histogram of mortality rates in one year’s time for US 65-year-old males based on cross-sectional historical simulation
scenarios but for just one future date. This is called cross-sectional historical simulation (see Figure 8.2) and involves applying all the historical relative changes in rates to the current mortality rate. For example:

\[ z_{2006,1} = \exp \left( r_1 \right) \cdot q_{2005} \]
\[ z_{2006,2} = \exp \left( r_2 \right) \cdot q_{2005} \]
\[ z_{2006,3} = \exp \left( r_3 \right) \cdot q_{2005} \]

etc.

This procedure generates 40 scenario paths for this mortality rate at one future date in one year’s time: \( z_{2006,1}, z_{2006,2}, \ldots, z_{2006,40} \).

Figure 8.3 and 8.4 show the results of historical simulation applied to the mortality rate for 65-year-old males in England & Wales. Based on data from 1961-2005, Figure 8.3 illustrates longitudinal historical simulation over the next 44 years. Figure 8.4 uses a histogram to reflect the result of cross-sectional historical simulation, which has generated 44 different scenarios for the mortality rate of this group in one year’s time. Figures 8.5 and 8.6 show the same results for US data.

Historical simulation is a simple and transparent technique for simulating future mortality rates. It is particularly useful in short term (1-year) simulations to capture volatility in mortality rates, but it has its limitations. In particular, it cannot account for jump risk, unless such a jump has occurred in the historical data set, and it assumes that historical volatility will be repeated. Moreover, for long-term simulations the method only produces one path (or at best a small number of slightly shorter paths) and, therefore, does not permit the calculation of long-run statistical risk metrics.

8.4.4.4 Other stochastic simulation models

In Chapter 7 we gave an overview of a number of different stochastic mortality models that can be used to produce simulations of possible future paths for mortality rates over the long term. These paths enable us to make an assessment not only of the expected level of mortality rates far into the future, but also of the risk associated with those rates. As an example, Figure 8.7 shows a set of paths for a 65-year-old male generated by a Bayesian Lee-Carter model developed by JPMorgan. The dispersion in the paths illustrates the magnitude of the risk to future mortality rates.

8.4.4.5 Using simulation models

It is important to note that, as they stand, the simulation models described here and in Chapter 7 only simulate mortality rates. To be useful for risk management these mortality scenario paths first need to be combined with scenario paths for interest rates and inflation (if exposures are inflation-linked). Furthermore, if the asset side of the balance sheet is also to be considered, then the investment returns must also be simulated. This enlarged set of paths must then be applied to the exposures themselves so that the cash flow profile of the exposure and its value in each scenario may be determined. From the resulting set of paths for cash flow and value, we can then calculate the expected value, expected cash flow and risk associated with the exposure.

Finally, it is worth reiterating the warning given in Chapter 7 and in Cairns et al. (2007). Different models have different strengths and weaknesses. It is important that the simulation model that is chosen for a particular situation is appropriate, which means making a judgement involving not just goodness-of-fit, but also balancing transparency, objectivity and parsimony.
8.5 Case study: Evaluating longevity exposure for a DB pension plan

In this section we evaluate the longevity exposure for a DB pension plan by first examining the pension for a single cohort of beneficiaries and then the pension plan as a whole.

8.5.1 Pension for a single cohort

The pension for a single cohort is effectively a portfolio of annuities for annuitants of the same age (ignoring spouse, dependant and death benefits). Here we consider the pension for a cohort of 65-year-old males, who were born in 1941.

Table 8.2 lists the key longevity-related statistics for 65-year-old males taken from the LifeMetrics Index. It shows both historic and current values for graduated initial rates of mortality, period life expectancy (i.e., assuming no mortality improvements), the value of a fixed pension and the value of an inflation-linked pension. Throughout this section pension valuations are based on the relevant period life table corresponding to graduated initial rates of mortality, without improvements. The valuations also use a constant uniform interest rate of 5% and a constant uniform inflation rate of 3%, so that we can focus on the impact of longevity, without being distracted by interest rate risk and inflation risk.

Note that over the period shown in Table 8.2, mortality for US males aged 65 improved by 51% and life expectancy increased by 4.31 years. The resulting change in the value of a fixed pension for these individuals was 25% and for an inflation-linked pension 30%. For England & Wales the figures are similar: mortality rates improved 59%, life expectancy increased 5.19 years and pension values increased by 32% (for a fixed pension) and 40% (for an inflation-linked pension).

8.5.1.1 Impact of volatility on pension value

The cross-sectional historical simulation of mortality rates we performed in the previous section can be used to ascertain the volatility in pension values for our cohort of 65-year-old males. Figures 8.8 and 8.9 show the distribution of values for a fixed pension in one year’s time for both England & Wales and the US. Note that these distributions are based on historical one-year mortality movements in each country and lead to very different shapes of the distributions.

8.5.1.2 Impact of mortality improvements on cash flows & values

So far we have assumed zero improvement in mortality rates. Figures 8.10 and 8.11 show the best estimate cash flows for a fixed pension for both the England & Wales and US cohorts. The cash flows in the figures assume 0% improvement.
Table 8.2 Historical changes in mortality rates, period life expectancy and pension value for 65-year-old males

<table>
<thead>
<tr>
<th>England &amp; Wales</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mortality rate (q)</td>
<td>Period life expectancy</td>
</tr>
<tr>
<td>1961</td>
<td>3.71%</td>
</tr>
<tr>
<td>2005</td>
<td>1.52%</td>
</tr>
<tr>
<td>Change</td>
<td>-58%</td>
</tr>
</tbody>
</table>

Table 8.3 Change in life expectancy and pension value for a 1% compounded annual improvement in mortality

<table>
<thead>
<tr>
<th>England &amp; Wales</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Males</td>
<td>65</td>
</tr>
<tr>
<td>Change in period life expectancy</td>
<td>1.01</td>
</tr>
<tr>
<td>Change in value of pension</td>
<td>3.31%</td>
</tr>
</tbody>
</table>

Figure 8.10 Cash flows for a fixed pension provided to a cohort of 65-year-old EW males

Figure 8.11 Cash flows for a fixed pension provided to a cohort of 65-year-old US males

Figure 8.12 Best estimate liability cash flows for a US defined benefit pension plan

Figure 8.13 The increase in liability cash flow for a US DB pension plan, resulting from a 1% compounded annual improvement in mortality rates
(no change) and 1% compounded annual improvement in mortality rates. Note that the meaning of a 1% compounded annual improvement in mortality rates is that all of next year’s mortality rates will be 99% of today's rates, the mortality rates for the year after next will be \( 99\% \times 99\% = 98.01\% \) of today's rates, etc.

The impact of a 1% compounded annual improvement in mortality rates is an increase of 3.57% in the value of the US pension and 3.31% in the value of an England & Wales pension. (Recall that these figures only apply to 65-year-old males and will differ for females and other ages).

In any longevity risk analysis it is essential to have a good understanding of the sensitivities of the exposure to mortality improvements and to increases in life expectancy. Table 8.3 shows the impact of a 1% compounded annual improvement in all mortality rates on both period life expectancy and pension valuation for 65 and 75 year olds. This provides an important intuitive linkage between these three metrics (mortality, life expectation, value).

8.5.2 Longevity risk of a pension plan

Having analyzed in the previous section the longevity risk associated with the pension of a single cohort, we now extend this to an entire pension plan comprised of multiple cohorts as well as males and females. For simplicity we assume that the plan contains only deferred members and retired members (pensioners).

The total value of the pension liabilities for this plan is $508 million and best estimate cash flows for this pension plan are shown in Figure 8.12. The deferrals represent 32% of the value, have an average age of 52 years and an interest-rate duration of 15 years. By contrast the pensioners represent 68% by value, with an average of 73 years and an interest-rate duration of 7 years.

8.5.2.1 Impact of mortality improvements on cash flows & values

Figure 8.13 shows the increase in liability cash flows for this plan resulting from a 1% compounded annual improvement in mortality. This mortality improvement leads to an increase in the total value of liabilities of 3.9%, with the value of retiree liabilities (pensions in payment) increasing by 3.3% and deferred liabilities increasing by 5.2%. This impact on value is illustrated in Figure 8.14.

The above analysis assumes that the pension plan pays a fixed benefit to members, but it is instructive to compare the results against an inflation-linked benefit. The impact of a 1% compounded annual mortality improvement on an identical inflation-linked pension plan is significantly higher. Overall, plan liabilities rise by 5.9%, with deferred liabilities rising 8.4% and retiree liabilities rising 4.7%. This is shown in Figure 8.15.

This case study has illustrated the methods used to evaluate the level of mortality and longevity risk associated with...
a particular DB pension liability. Of particular note is the much greater sensitivity of (i) deferred pensions and (ii) inflation-linked pensions to mortality trend risk. So we may conclude that deferred, inflation-linked pensions are the most sensitive to long-term longevity risk, while retiree, fixed pensions are least sensitive to long-term longevity risk.

Note that these same methods can be applied with equal effectiveness to other types of life-contingent exposures.

8.6 Conclusion

In this chapter we have described two key steps in the LifeMetrics Framework connected with understanding the nature of the exposure and evaluating the impact of longevity and mortality risk on the value and cash flows of the exposure. The final remaining step in the framework is to use this information to decide on a course of action, such as to hedge, transfer or retain the exposure. This step is addressed in the next chapter.

Table 8.4 Mortality catastrophe bond issues

<table>
<thead>
<tr>
<th>Issue</th>
<th>Issuer</th>
<th>Size</th>
<th>Issue date</th>
<th>Tenor</th>
<th>Attachment</th>
<th>Exhaustion</th>
<th>Investor cpn</th>
<th>Rating</th>
<th>Underlying Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vita Capital I</td>
<td>SwissRe</td>
<td>$400mm</td>
<td>Dec 2003</td>
<td>3yr</td>
<td>130%</td>
<td>150%</td>
<td>L + 135bps</td>
<td>A+/A3</td>
<td>US (70%), UK (15%), France (7.5%), Italy (5%), Switzerland (2.5%)</td>
</tr>
<tr>
<td>Vita Capital II</td>
<td>SwissRe</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Class B</td>
<td></td>
<td>$62mm</td>
<td>Apr 2005</td>
<td>5yr</td>
<td>120%</td>
<td>125%</td>
<td>L + 90bps</td>
<td>A/Aa3</td>
<td>US (62.5%), UK (17.5%), Germany (7.5%), Japan (7.5%) and Canada (5%)</td>
</tr>
<tr>
<td>Class C</td>
<td></td>
<td>$200mm</td>
<td>Apr 2005</td>
<td>5yr</td>
<td>115%</td>
<td>120%</td>
<td>L + 140bps</td>
<td>A/A2</td>
<td></td>
</tr>
<tr>
<td>Class D</td>
<td></td>
<td>$100mm</td>
<td>Apr 2005</td>
<td>5yr</td>
<td>110%</td>
<td>115%</td>
<td>L + 190bps</td>
<td>BBB/Baa2</td>
<td></td>
</tr>
<tr>
<td>Tartan Capital</td>
<td>ScottishRe</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Class A</td>
<td></td>
<td>$75mm</td>
<td>May 2006</td>
<td>3yr</td>
<td>115%</td>
<td>120%</td>
<td>L + 19bps</td>
<td>AAA/Aaa</td>
<td>US</td>
</tr>
<tr>
<td>Class B</td>
<td></td>
<td>$80mm</td>
<td>May 2006</td>
<td>3yr</td>
<td>110%</td>
<td>115%</td>
<td>L + 300bps</td>
<td>BBB/Baa3</td>
<td></td>
</tr>
<tr>
<td>Osiris Capital</td>
<td>Axa</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>4yr</td>
<td>114%</td>
<td>119%</td>
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Source: Bloomberg, Vita, Tartan and Osiris Offering Circulars, Investor Presentations. Latest Bloomberg data used for rating verification

L = Libor, E = Euribor
9. Hedging longevity and mortality risk

9.1 Introduction

Having described the framework and some of the key tools for measuring the impact of longevity and mortality risk on pensions, annuities and other exposures in the previous chapter, this chapter addresses the hedging and transfer of this risk. This is the fifth and final step in the LifeMetrics Framework that we outlined in Chapter 2.

Hedging is a decision that requires an assessment of the trade-off between the benefits of reducing risk and the costs of implementing the hedge. The benefits of hedging can manifest themselves in many ways: lower likelihood of financial distress or insolvency, lower capital requirement, higher credit quality, increased debt capacity, greater flexibility, and/or an ability to take an equivalent amount of risk elsewhere for a greater expected return. The costs of hedging fall into two categories: direct costs and opportunity costs. For example, a pension plan that hedges its interest rate risk with interest rate swaps pays the (small) direct transactional cost of the swap, but also faces a (larger) opportunity cost if interest rates rise.

In order to be able to assess this trade-off it is necessary to evaluate the effectiveness of any proposed hedge in reducing, or eliminating the risk. Evaluating hedge effectiveness involves analyzing the risk characteristics of both the exposure being hedged and the hedging instrument. In particular, it is important to understand the degree to which they are correlated, or cointegrated, with each other so that the residual risk can be quantified. For longevity and mortality hedges, one important residual risk can be the basis risk between the population relating to the underlying exposure and the population relating to the hedge.

We begin in the next section by introducing the subject of hedge effectiveness. As well as outlining the basic principles, we describe a framework for assessing effectiveness that is equally applicable to hedges of mortality and longevity as it is to hedges of other types of risk. In Section 9.3 we take up the subject of population basis risk. After discussing the various factors that characterize it, we examine some examples of age-related basis risk, and then present two case studies assessing the long-term effectiveness of hedges based on the LifeMetrics Index. In Section 9.4 we address the advantages and disadvantages of standardized hedges that reference a national index versus customized hedges that reference the specific population of the exposure. In particular, we describe an approach to hedging that uses standardized building-blocks that combine flexibility and hedge effectiveness with the potential for high liquidity. Finally in Sections 9.5 and 9.6, we present two hedging case studies: one addressing longevity hedging for a pension plan and the other examining the degree of risk offset between an insurer’s life portfolio and annuity portfolio.

9.2 Principles of hedge effectiveness

Intuitively, hedge effectiveness reflects how well a hedging instrument protects an underlying exposure from undesirable changes in performance arising from a particular risk. In this section we discuss the key conceptual and practical aspects of hedge effectiveness, and provide a framework for implementing appropriate effectiveness tests for mortality and longevity. The concepts discussed in this section can be found in Coughlan et al. (2003, 2004) and Coughlan (2004).

9.2.1 Definition

Hedge effectiveness can formally be defined as the degree to which changes in the performance of an exposure (the “hedged item” or “underlying”) in respect of a designated risk are offset by changes in the performance of a designated hedging instrument.

This definition emphasizes that the effectiveness of any hedge must be evaluated in terms of hedging objectives that include:

- The risk that is being hedged
- The performance metric

In practice an exposure will be subject to several different risk factors, such as interest rate risk, inflation risk, investment risk, longevity risk, etc. Because these risks interact with each other in a non-linear way, it is important that
hedge effectiveness assessments focus purely on the particular risk designated as the object of the hedge, and exclude the effects of other risks. So if longevity risk is the designated risk, then the impact of interest-rate risk and other risks should be excluded from the hedge effectiveness assessment.

The performance metric refers to the quantity that will be used to evaluate the effectiveness of the hedge, for example cash flow or market value. If the relevant performance metric is cash flow then effectiveness needs to be evaluated in terms of hedging the impact of the risk on cash flow. Similarly if the metric is value then effectiveness must be defined in terms of the impact of the risk on value.

9.2.2 The “perfect hedge”

When evaluating a hedging relationship it can be useful to measure the effectiveness of a particular hedge relative to a so-called “perfect hedge”. A perfect hedge is a hedge that is completely effective in eliminating the impact of designated risk on the chosen performance metric over the life of the hedging relationship. In other words, changes in the performance of the hedging instrument precisely offset the changes in the performance of the underlying hedged item, in respect of the designated risk.

In reality, the majority of hedges that are implemented by any organization are not perfect hedges, but nevertheless play an important role in reducing risk. As long as the hedging instrument provides a material level of economic offset between its performance and the performance of the underlying hedged item, with respect to the designated risk, the hedge should be deemed effective.

9.2.3 Evaluating hedge effectiveness

The most direct way of measuring hedge effectiveness is in terms of the amount of risk reduction (or the amount of offset) achieved through the hedging relationship. This involves comparing the risk associated with the underlying exposure against the portfolio formed by the combination of the exposure and the hedging instrument. The effectiveness result will depend on the risk characteristics of the exposure, the risk characteristics of the hedging instrument and the correlation between them.

Let $U$ represent the underlying exposure (hedged item) and $H$ represent the hedging instrument. Then the portfolio $P$ consisting of a combination of the two is given by:

$$P = U + hH$$

where $h$ is the so-called “hedge ratio”, i.e., the amount of the hedging instrument that is used to hedge one unit of the hedged item. Hedge effectiveness can be measured in terms of the amount of risk reduction in the following way:

$$R = 1 - \frac{\text{Risk}(P)}{\text{Risk}(U)}$$

If the hedge is perfect, then the risk associated with the portfolio is zero and the risk reduction is 100%. Generally, however, a hedge will not be perfectly effective and the risk reduction will often be less than 100%. Nevertheless hedge effectiveness can be optimized by selecting the hedge ratio $h$ to achieve the lowest risk portfolio, or in other words, the maximal risk reduction.

Example

It can be shown that if risk is measured in terms of volatility (i.e., standard deviation) and changes in the relevant performance metric are normally distributed, then the optimal hedge ratio, denoted $h^*$, is given by

$$h^* = -\rho \left( \frac{\sigma_U}{\sigma_H} \right)$$

where $\sigma(U)$ and $\sigma(H)$ are the volatilities of the underlying exposure and the hedge respectively and $\rho$ is the correlation between them. With this hedge ratio, the optimal risk reduction depends only on the correlation between the exposure and the hedge:

$$R^* = 1 - \sqrt{1 - \rho^2}$$

If the hedge is perfect then the correlation between the hedging instrument and the exposure is -1, so that the optimal hedge ratio is just the ratio of the volatilities and the corresponding level of risk reduction is 100%.

9.2.4 Framework for hedge effectiveness

In 2003 a framework for hedge effectiveness, called HEAT (Hedge Effectiveness Analysis Toolkit), was published.
(Coughlan et al., 2003, 2004). This framework sets out five steps for evaluating hedge effectiveness:

- Step 1: Define hedging objectives
- Step 2: Select hedging instrument
- Step 3: Select methodology for hedge effectiveness evaluation
- Step 4: Evaluate effectiveness
- Step 5: Interpret the effectiveness results

In defining hedging objectives in step one, it is important to carefully specify the designated risk being hedged. This includes the risk class (e.g., longevity risk, or mortality risk), as well as the precise nature of what is being hedged (e.g., longevity trend improvements above 2% per year over the next 10 years, or mortality jump risk above 130% over a single year). Clearly, an essential part of the objectives should also be the time horizon of the hedging relationship and the performance metric.

Once the hedging instrument has been selected in step two, the optimal hedge ratio should be chosen to maximize the degree of risk reduction.

The third step - the definition of the hedge effectiveness methodology - is important, because an inappropriate choice can lead to spurious and misleading results with effective hedges being deemed ineffective. The methodology for hedge effectiveness involves seven distinct elements:

1. Reference exposure: Defines what the hedge should be compared with in the effectiveness test. Obvious choices are the underlying exposure, or the perfect hedge.
2. Approach to measurement: Specifies how the impact of the designated risk on the chosen metric is evaluated. For example, evaluation of changes in value or cash flow due to changing mortality and longevity may assume that interest rates are unchanged and fixed at current levels.
3. Historical data set: Defines the historical data to be used for hedge effectiveness evaluation. It includes making a choice about how much history, from what periods and with what frequency.
4. Simulation method: Specifies how historical data can be used to make a prospective assessment of hedge effectiveness. It involves specifying stress tests or stochastic models that will be used to simulate the performance of the underlying exposure and the hedged item in the future.
5. Maturity treatment: Defines whether the effectiveness assessment is performed until maturity (or runoff), or evaluated over a shorter horizon.
6. Basis for comparison: Specifies whether the hedge effectiveness evaluation will be based on interim and/or final comparison of performance. If interim, will it be on a cumulative or period-by-period basis?
7. Type of effectiveness test: Defines whether the test is a risk reduction test, or a regression, or a simple offset calculation under specific scenarios, etc.

Step four in the HEAT framework involves implementing the effectiveness calculation specified in the methodology step. This means first performing a simulation to produce scenario paths and/or developing stress tests for mortality rates, interest rates, etc. Second, both the underlying and the hedging instrument should be evaluated in terms of the chosen performance metric under the scenarios and/or stress tests. Finally the impact of the risk on the underlying and the hedging instrument with respect to the chosen metric is used to evaluate the amount of risk reduction, or degree of offset.

Step five involves interpreting the results and making a judgment on whether the level of offset or the level of risk reduction is sufficient for the hedge to be deemed effective.

9.3 Sources of hedge ineffectiveness: Population basis risk

9.3.1 Introduction

There is no obvious way to hedge all the mortality and longevity risk associated with an exposure unless the hedge is customized to the underlying population. Only then could it be a perfect hedge. Nevertheless, for a hedge based on a benchmark population, such as national population, it is still generally possible to eliminate a meaningful portion of the mortality and longevity risk. If the hedge is carefully constructed to match the initial sensitivity of the underly-
ing to mortality risk, and periodically rebalanced, then the 
mortality risk that remains (basis risk) can be reduced to 
two components relating to:

- Statistical sampling error for small populations
- The different risk characteristics of the national popula-
tion vs. the underlying population.

While the sampling error component is material for a small 
population, it is unlikely to be an issue with a large popula-
tion. The other component relating to differences in risk 
characteristics between the populations is more interest-
ing as it is a source of basis risk that can be minimized 
depending on the precise nature of these differences and 
how well the hedge is structured.

9.3.2 Characterizing basis risk

There are several potential sources of hedge ineffectiveness 
arising from population basis risk. Recall that basis risk 
relates to the difference in mortality experience between 
two different populations. In a hedging situation it arises 
from the mismatch between the specific population associ-
ated with the underlying exposure and the population as-
associated with the hedging instrument (e.g., a hedge that 
references a national population index such as LifeMetrics). 
Basis risk can be classified in terms of mismatches between 
one or more of the following population characteristics:

- Age profile
- Gender profile
- Marital status
- Socio-economic profile
- Lifestyle profile
- Health profile
- Geographical profile
- Etc.

If two populations have different profiles then they will 
generally have different longevity and mortality experiences. 
These differences may or may not be material when trans-
lated into their impact on value or cash flow. Materiality 
depends on the specific nature of populations, as well as 
the characteristics of the exposure itself and the hedging 
instrument.

9.3.3 Age profile mismatch

In this subsection we address a particular form of basis risk 
related to mismatches in age profiles between populations. 
As we saw from the historical movements in the LifeMetrics 
Index in Chapter 6, the mortality curve does not always 
shift in parallel, or even proportionately across ages. This is 
further emphasized in Figures 9.1 and 9.2, which show rela-
tive shifts in the mortality curve over five-year periods for 
females in England & Wales and the US. Similar results hold 
for males. The shifts have been very different at different 
ages, but they have been relatively smooth across ages. In 
particular, adjacent ages have experienced shifts of a simi-
lar magnitude.

One can think of many reasons why it is reasonable for the 
mortality curve to shift in complex ways, with the young 
end of the curve moving differently from the old end. For 
example, when AIDS became widespread it had a more ma-

Figure 9.1 Historical 5-year movements in the mortality 
curve for EW females (%)

Figure 9.2 Historical 5-year movements in the mortality 
curve for US females (%)

-20% -10% 0% 10% 20% 30%
0 50 60 70 80 90 Age

-20% -10% 0% 10% 20% 30%
0 50 60 70 80 90 Age
terial impact on the mortality of younger adults compared to the elderly, leading to a “flattening” of the mortality curve. On the other hand, a very hot summer, or very cold winter, is more likely to increase mortality rates for the elderly relative to younger members of the population, leading to a “steepening” of the curve across age.

Intuitively there is little reason to expect that the mortality improvements for one age would not be highly correlated with those for other nearby ages. However, with raw mortality data, these correlations are generally lower than intuition might suggest, particularly over short periods such as a year. The major reason for this lies in the noise associated with raw mortality rates. If, instead, one considers graduated mortality rates the correlations in mortality movements between neighboring ages are much higher and more in line with intuition. Similarly, if instead of graduated rates, one considers raw rates grouped into, say, five-year age buckets, then again one finds higher, more intuitive correlations between adjacent buckets.

Figure 9.3 shows historical correlations in the movements of graduated initial rates of mortality between 45 year-old males and each individual age group between 40 and 50, using annual data from England & Wales over the period 1961-2005. Note that as expected, the correlations are higher nearer to age 45 and fall off gradually on either side as the age gap widens. Figures 9.4 to 9.6 show the same correlation analysis for ages 55, 65 and 75 for England & Wales. Note that the correlations between adjacent ages increase with age so that the correlation between a 45-year-old and a 50-year-old is 43%, while that between a 65-year-old and a 70-year-old is 90%. Similar patterns can be observed in the female population.

Figures 9.7 to 9.10 show the same analysis for US males, where it can be seen that a similar result emerges.

Note that this correlation analysis is based on mortality changes over a one-year period, which, despite being based on graduated mortality rates, still contain noise. (This is because graduation of mortality rates in the LifeMetrics Index only reduces the noise across ages, not across time). So repeating the correlation analyses for changes over longer periods generally results in higher correlations, because the impact of noise is reduced over longer horizons. However, longer periods over which to measure the change in mortality also mean fewer data points and, therefore, a higher statistical uncertainty in the correlation estimates. This increased sampling error might lead to the expected increase in correlation not materializing for certain specific ages, simply because only a few data points are used and the correlation result can be spurious. Despite this, the result is generally more reliable at higher ages.

One of the important conclusions to be drawn from this correlation analysis is that mismatches in age profile between two populations do not necessarily lead to significant basis risk between the populations, particularly at high ages. As long as the mismatch in age profiles reflects age differences of only a few years apart, then the impact will be small. This is an important result to bear in mind in the design of hedges and in the assessment of hedge effectiveness.

9.3.4 Impact of population basis risk

Regardless of whether the source of basis risk lies in mismatches between the profiles of ages, socioeconomic status, geography, etc., it can be summarized in terms of:

- The differences in base mortality rates for the two populations
- The differences in the way mortality rates will change through time for the two populations

The first of these refers to mismatches in the current levels of mortality rates and in the shapes of the current mortality curves. The second of these refers to mismatches in the profiles of future movements in mortality rates (improvements or deteriorations).

Suppose we are contemplating a hedge to lock in the value of an exposure with respect to longevity and mortality risk at a future date. What is important from a hedge effectiveness perspective is the degree to which changes in the value of the exposure are offset by changes in the value of the hedging instrument. So the first step in designing an effective hedge is to quantify how differences between the two populations in terms of base mortality rates and future movements in rates affect the value of the exposure relative to the hedging instrument. Then the second step
Figure 9.3 EW mortality improvement correlations for males aged 45

Figure 9.4 EW mortality improvement correlations for males aged 55

Figure 9.5 EW mortality improvement correlations for males aged 65

Figure 9.6 EW mortality improvement correlations for males aged 75

Figure 9.7 US mortality improvement correlations for males aged 45

Figure 9.8 US mortality improvement correlations for males aged 55

Figure 9.9 US mortality improvement correlations for males aged 65

Figure 9.10 US mortality improvement correlations for males aged 75
is to structure the hedging instrument to minimize the impact of these differences on relative changes in value.

Base mortality rates for two populations can differ significantly, and a hedging instrument would need to match the initial sensitivity of the underlying as closely as possible. In addition, movements in mortality rates through time need to be addressed. To address these movements we have analyzed the way mortality rates have changed through time for different populations, assuming they have the same (or similar) current mortality. In the first such analysis we compare the England & Wales overall population with a population of individuals who have a life insurance policy. Clearly, this analysis addresses the basis risk between insured lives and the population at large. In the second analysis we compare the US national population with the population of the state of California.

### 9.3.5 Case study: LifeMetrics Index vs. insured lives

In this example, the underlying exposure consists of a portfolio of fixed pension liabilities (or equivalently annuities) being paid to a large population of 65-year-old males, all of whom live in England & Wales. The mortality experience of the population is defined by the historical data collected by the CMIB for insured lives in the UK. Generally, insured lives experience lighter (i.e., lower) mortality than the general population.

The hedging instrument is a synthetic annuity derivative structured to have (approximately) the same base (current) mortality profile as the underlying population. However, the future mortality experience of the hedging instrument is linked to the movements in mortality rates for the population of England & Wales (as reflected in the corresponding LifeMetrics Index), while the underlying population experiences its own mortality movements. The hedging instrument is identical to the underlying exposure, except for two things. It has the opposite sign and its future mortality experience is linked to a different population.

Assuming constant interest rates, Figures 9.11 to 9.14 show the evolution in value of the underlying and the value of the hedging instrument over a ten-year horizon, based on historical mortality movements from four different ten-year periods.

Figure 9.11 shows the effect of employing such a hedge on the underlying portfolio using historical data from the period 1961-1971. Essentially, the two differ only because the hedging instrument experiences mortality improvements reflecting the England & Wales population whereas the underlying exposure experiences mortality improvements reflecting the CMIB’s insured lives population. The chart clearly shows that, despite some year-on-year variability between the underlying exposure and the hedging instrument, over the long ten-year horizon their trends are very similar. At the end of the 10-year period the ratio of the hedge value to the underlying value is 97.7%.

Figures 9.12 to 9.14 reflect identical analyses but applied to mortality improvement data from different historical periods. They all show that the values of the hedging instrument and the underlying exposure do not deviate too far from each other over the horizon, with the ratio staying close to 100%.

The conclusion we draw from this analysis is that despite the very different populations, the historical movements in mortality do not lead to a significant amount of residual risk over any of the ten-year periods. In other words, the mortality trend improvement for the different populations is strongly correlated, and a hedge that is designed to have the same initial sensitivity as the underlying would indeed be an effective hedge for mortality improvement, apart from events not reflected in the historical mortality data.

### 9.3.6 Case study: LifeMetrics Index vs. state population

A similar example to that discussed above was applied to US data, where mortality improvements for the underlying exposure were based on the mortality experience of the California state population and those for the hedging instrument were based on the mortality experience of the US national population (reflected in the LifeMetrics Index). The results are shown in Figures 9.15 and 9.16 and are completely consistent with the England & Wales analysis above. In this example, two historical 10-year periods were used to assess the basis risk of mortality improvements between the US national population and California state population. The ratios of the value of the hedging instrument to that of the underlying exposure remain very close to 100% through-
Figure 9.11 Pension value EW cohort 65yr males 1961-71

Figure 9.12 Pension value EW cohort 65yr males 1971-81

Figure 9.13 Pension value EW cohort 65yr males 1981-91

Figure 9.14 Pension value EW cohort 65yr males 1991-2001

Figure 9.15 Pension value US cohort 65yr males 1980-90

Figure 9.16 Pension value US cohort 65yr males 1990-2000
out each period, with the end-of-period ratios being 98.5% and 99.0% respectively.

The conclusion we draw from this analysis is the same conclusion we drew from the previous example. In particular, despite the different populations, historical movements in mortality do not lead to a significant amount of residual risk over either of the ten-year periods. In other words, this particular structure for the hedging instrument has minimized the impact of the basis risk of mortality improvements on the hedge effectiveness result. So, apart from events not reflected in the historical mortality data, and provided the underlying population is large enough, this hedge based on national data with reference to the LifeMetrics Index should be effective.

9.4 Structuring effective and liquid hedges

9.4.1 Standardized vs. customized hedges

In any hedging situation, there is a tension between pursuing a hedge customized to the precise risk characteristics of the hedged item and a more standardized, general purpose hedge. Hedging longevity and mortality risk is no exception. Both customized and standardized hedges have their advantages and disadvantages, which basically boil down to a trade-off between perfect hedge effectiveness vs. liquidity and cost. These pros and cons are discussed in more detail below, but first let us explain what customized and standardized hedges of longevity and mortality risk might look like.

A customized longevity/mortality hedge would be tailored to reflect the actual longevity/mortality experience of the specific population in question, so as to completely eliminate the risk. It would be structured as a cash flow hedge so that the net cash flow (exposure cash flow + hedge cash flow) is fixed with respect to changes in longevity/mortality. The maturity of the customized hedge would be such that it continues until the runoff of the last liability payment.

By contrast, a standardized hedge would be based on the longevity/mortality experience of the national population, but calibrated to match the sensitivity of the actual population to changes in mortality rates. It would generally be structured as a hedge of value, rather than a hedge of cash flow, so that any increase in the value of liabilities due to changes in mortality would be offset by a compensating payment provided by the hedge. The maturity of a standardized hedge might typically be 5, 10 or 20 years — much shorter than that of the customized hedge.

Advantages and disadvantages

The advantages and disadvantages of standardized hedges and customized hedges are summarized in Table 9.1. While the customized hedge provides complete risk mitigation, it is likely to be more costly, more credit intensive and more

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<th>Advantages</th>
<th>Disadvantages</th>
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<td>Standardized Hedge</td>
<td>Cheaper than customized hedge</td>
<td>Not a perfect hedge</td>
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<td></td>
<td>Lower set-up / operational costs</td>
<td>- Residual basis risk</td>
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<td>Potentially to be liquid</td>
<td>- Roll risk at maturity</td>
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<td></td>
<td>Shorter maturity</td>
<td>Needs periodic rebalancing</td>
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<td></td>
<td>- Lower counterparty credit exposure</td>
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<tr>
<td>Customized Hedge</td>
<td>Can be an exact hedge</td>
<td>More costly than standardized hedge</td>
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<td></td>
<td>- No residual basis risk</td>
<td>Higher set-up / operational costs</td>
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<td>Set-and-forget hedge requires minimal monitoring</td>
<td>Poor liquidity</td>
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<td>- Difficult to adjust or unwind</td>
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<td>Longer maturity</td>
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<td>- Higher counterparty credit exposure</td>
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cumbersome to adjust or unwind. Furthermore, depending on the legal and regulatory jurisdiction it may not even be available as an alternative. The standardized hedge, on the other hand, is likely to be more economical, less complex and much more liquid. The key disadvantage is that it does not completely eliminate the longevity/mortality risk and may require periodic rebalancing. In particular, there will be a residual exposure to population basis risk; however, this basis risk can be minimized through careful construction of the hedge and periodic rebalancing.

Standardized risk-transfer and hedging instruments are an important prerequisite for the development of a liquid traded market for longevity and mortality.

Despite the fact that standardized hedging instruments can be highly effective hedges, they are not perfect hedges. As a result, a hedge effectiveness assessment should always be carried out to ascertain the level of residual risk.

9.4.2 Standardized building-block hedges

9.4.2.1 Combining flexibility, liquidity and effectiveness

JPMorgan has developed a unique approach to hedging longevity and mortality risk that involves a set of standardized building-block hedges, based on national population data. This approach combines the advantages of standardization outlined earlier in this chapter (liquidity, fungibility, cost, etc.) together with the flexibility of a building-block approach needed to design effective hedges for different types of exposures.

Using this approach it is possible to structure a hedge so as to minimize the effect of differences in current (or base) mortality rates, which means that for a large enough population the primary form of basis risk will be that coming from the mismatch in mortality movements between the specific population associated with the underlying exposure and that of the national population. By matching the overall sensitivity of the value of the exposure and the value of the hedging instrument to the mortality movements for the national population, we can maximize the effectiveness of this hedge.

9.4.2.2 q-duration

To structure this hedge appropriately we have developed a technique for measuring the mortality sensitivity of any underlying exposure. By analogy with the concept of interest rate duration, we call this sensitivity “q-duration”, which is defined as the change in value of a portfolio due to a unit change in mortality rates. From the q-duration of the portfolio we create a portfolio of offsetting, standardized building-block hedges based on payoffs which are a function of the LifeMetrics index.

Note that this is not possible with a single benchmark hedging instrument, such as a hedge based on life expectancy, because the flexibility to adjust the notional of each of the building-block hedges is needed in order to maximize hedge effectiveness.

9.5 Case study: Pension plan longevity hedging

In this section we walk through a case study of how a defined benefit pension plan can reduce the longevity risk in its liabilities through a set of standardized, building-block hedges.

9.5.1 Sensitivity of plan to changes in mortality rates

The value of a pension plan’s liabilities is the present value of future expected cash flows, based on assumed mortality rates. As such, this value is subject to longevity risk, i.e., the risk that beneficiaries live longer than expected, and

<table>
<thead>
<tr>
<th>Table 9.2 Value of the pension plan liabilities in ten years time</th>
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<tbody>
<tr>
<td>Using current mortality assumptions</td>
<td>$523mm</td>
</tr>
<tr>
<td>Assuming an additional 1% annual improvement in mortality</td>
<td>$556mm</td>
</tr>
<tr>
<td>Difference</td>
<td>$33mm</td>
</tr>
</tbody>
</table>
pension benefits need to be paid out longer than expected. An increase in longevity (corresponding to a decrease in mortality rates) increases the future expected cash flows and therefore the present value of the liabilities.

Let us consider the hypothetical case of a pension plan whose sponsor is concerned about the uncertainty in the future value of its liabilities. In Table 9.2, we illustrate the potential value of those liabilities in ten years’ time, using the current best estimate mortality assumptions for the plan. We also estimate the value of those liabilities after an additional 1% compounded annual improvement in mortality rates beyond these best estimate expectations. These figures provide an indication of the potential uncertainty in the value of the plan liabilities in 2017, based on today’s estimate of the way the plan is likely to evolve and an adverse scenario that was used as a reasonable stress test. The potential increase in liability value in 2017 is approximately $33mm or 6.3%. In order to hedge the sensitivity of the liability value to mortality rates, the pension plan requires a hedging instrument, or a portfolio of hedging instruments, whose change in value offsets the potential change in value of the liabilities in virtually all conceivable mortality scenarios.

9.5.2 Hedging instruments

Through the use of derivatives, it is possible for a pension plan to hedge the risk associated with its liabilities without the need to effect a pension buyout or purchase annui-
ties. The use of derivatives, unlike a buyout or annuitization, avoids the loss of the related return-generating assets (although some assets will be required for collateral). Hedging the longevity risk of pension liabilities is a natural extension to hedging the interest rate risk or inflation risk of the liabilities. This suggests a general liability hedging strategy (commonly known as LDI or liability driven investing, although this term has lost its true meaning through overuse), in which “unrewarded” risks associated with the liabilities, such as interest rate risk, inflation risk and longevity risk are transferred out of the plan, permitting the risk budget to be reallocated more effectively. This is illustrated in Figure 9.17.

As discussed previously, a hedge of the pension longevity risk could be constructed as a customized hedge based on the exact benefit cash flows of the pension plan. However, here we consider a standardized format that hedges the movements in the future value of the liabilities over a specified horizon due to changes in mortality rates. The payoff of the hedge is referenced to the LifeMetrics Index and compensates the pension plan for any change in value of the liabilities over a given horizon (in this case 10 years) due to changes in mortality rates and changes in expectations about future mortality.

Through an analysis of the plan’s beneficiary data and historical experience relative to national mortality, the standardized hedge can be structured to eliminate most of the basis risk. This involves creating a set of standardized build-
ing-block hedges based on payoffs which are dependent on the LifeMetrics Index. The hedges and their notionals are chosen so as to match the combination of the initial mortality table and profile by age of the underlying pension liabilities in such a way that the structure has similar overall sensitivity to changes in mortality rates as the underlying pension plan’s liabilities.

9.5.3 Hedge Effectiveness

We have compared the set of standardized building-block hedges with a fixed-maturity customized 10-year hedge of the pension liabilities. The results of this comparison of relative effectiveness indicate that the standardized hedges, if properly calibrated, can provide similarly high levels of effectiveness (see Figure 9.18).

9.6 Case study: Combining life insurance and annuities

In this section we present a case study of the risk mitigation afforded by combining an insurer’s life book with an annuity book. This illustrates an application of the LifeMetrics Framework to integrating different kinds of mortality exposure. The characteristics of these exposures are summarized in Table 9.3. In this case study we have taken the liberty of assuming that we have reasonable influence on the balance of insurance and pension liabilities in our analysis. With a liquid market in mortality and longevity building blocks, this may be closer to reality than it is currently, as building-block hedges can be used to develop this balance.

As discussed in the previous chapter, annuities are exposed to longevity risk, meaning that the value of the liabilities increases as mortality rates fall. In principle, liabilities in respect of life insurance business have an offsetting risk to this, because life insurance liabilities decrease in value as mortality rates fall. Combining these two exposures together has the potential to result in a lower overall risk than either risk considered in isolation. This natural hedging of life and annuity risks has been tested empirically by Cox and Lin (2007).

However, the reality is not as simple as this for a number of reasons. First, the population associated with the life insurance portfolio may have a different demographic profile from the population associated with the annuity portfolio, because, for example, life insurance may be relatively more

Figure 9.18 Comparison of standardized and customized hedges over a 10-year horizon

![Figure 9.18 Comparison of standardized and customized hedges over a 10-year horizon](image)

Figure 9.19 Annuity book cash outflows

![Figure 9.19 Annuity book cash outflows](image)

Figure 9.20 Life book cash outflows

![Figure 9.20 Life book cash outflows](image)
concentrated amongst wealthier people than annuities. This goes to the heart of the topic of population basis risk that we have already discussed. Second, the average age of a life book may be lower than that of an annuity book, as individuals in retirement tend to have less of a need for life insurance. This leads to a further basis risk between the populations as discussed previously. Third, notwithstanding the above, the relative size of each portfolio will determine how much risk mitigation is possible, even if basis risk issues are minimized.

In this case study, we evaluate effectiveness of the life book and the annuity book as a hedge of each other’s exposure to mortality/longevity risk. This is done in two ways, in particular, by implementing:

- A deterministic stress test involving higher than expected improvements in mortality (i.e., a trend stress test)
- A statistical risk calculation based on value-at-risk (VaR)

### 9.6.1 Mortality stress test

The two charts in Figures 9.19 and 9.20 show the cash flow profiles for a sample annuity book and a sample life insurance book respectively. The base case cash flows are shown by the blue lines and the gray lines show the cash flows assuming 1% per annum additional compounded mortality improvements. For the annuity book the mortality improvements increase the cash flows whereas for the life book the improvements decrease the cash flows, which is as expected. The annuity book is made up of both fixed deferred annuities and fixed annuities in payment so these cash flows have about 60 years to run off. In contrast the life book is only made up of term assurances and all cash flows are paid out within the next 30 years.

Figure 9.21 shows the change in the value of each book caused by a 1% compounded annual improvement in mortality relative to best estimate. The blue bar in the chart shows that the change in value of the annuity book is -$20m when mortality improvements are 1% higher than the base case. On the right of this the gray bar shows the change in value on the life insurance book of $12m. The bar on the far right shows the change in value of the combined position.

### Table 9.3 Characteristics of the annuity book and life book

<table>
<thead>
<tr>
<th></th>
<th>Annuity book</th>
<th>Life Book</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of obligations ($mm)</td>
<td>508</td>
<td>165</td>
</tr>
<tr>
<td>Average age (yrs)</td>
<td>69.5</td>
<td>54.5</td>
</tr>
<tr>
<td>Interest rate duration (yrs)</td>
<td>9.4</td>
<td>7.9</td>
</tr>
</tbody>
</table>

### Figure 9.22 VaR for a one-year mortality simulation

![Figure 9.22 VaR for a one-year mortality simulation]

### Figure 9.23 VaR for a five-year mortality simulation

![Figure 9.23 VaR for a five-year mortality simulation]
of life and annuity liabilities. It is interesting to see that under this mortality scenario, the total change in value is only -$8m, less than the change in either the life book or the annuity book. Put another way, this result implies that the combined book has less sensitivity to mortality than either book in isolation.

In fact the level of risk reduction shown here is by no means the lowest risk attainable under this form of analysis. By choosing the appropriate size of the life book for a given annuity book it is possible to further reduce the impact on the value of the combined book.

**9.6.2 Mortality VaR analysis**

We can take this analysis a step further and look at the result when the improvement rates do not apply equally for all ages. This is carried out using historical simulation (as described in Chapter 8) by calculating the change in value of each exposure due to a number of different historical scenarios for mortality improvement, where each scenario reflects a movement in the mortality curve that is not uniform by age. The scenarios used are those corresponding to the LifeMetrics Index for England & Wales from 1961 to 2005. Note that the impact of these improvements is much smaller than for the 1% compounded annual improvement scenario.

From these scenarios we can compute a one-year Value-at-Risk (VaR) figure for each book at a 95% confidence level. This is estimated from the standard deviation of values under each scenario and multiplied by 1.65. (There are insufficient historical scenarios to calculate VaR directly). Figure 9.22 shows the VaR of the life book to be $6.13m and that of the annuity book to be $9.43m. Summing these together gives an undiversified VaR of $15.56m before allowing for any hedge benefit. But when the net VaR of the combined book is calculated this gives a total of just $7.45m - about half the simple sum. The net VaR is much lower than the sum of the VaRs because the impact of mortality movements on the two portfolios is partially offsetting.

Figure 9.23 involves the same VaR calculations but for five-year historical movements in mortality instead of one-year movements. The results are broadly similar except that the level of risk reduction is much more pronounced. This is mainly because correlations between ages tend to increase over longer horizons, as noise becomes less important relative to the size of the mortality change. Over the five year horizon, the sum of the stand-alone VaRs for each book is $24.35m, whereas the total VaR after allowing for offsets (hedging benefit) is $9.02m, close to a third of the sum of individual VaRs. Also worth noting is that the $9.02m is lower than either the VaR of the annuity book or the life book. The analysis suggests that a meaningful degree of risk reduction can be achieved between a life book and an annuity book, assuming the two populations are fairly similar demographically, even if there are material differences in their average ages.

Should an insurer wish to further reduce the residual risk between its life book and annuity book, it could take advantage of the standardized building-block hedges discussed earlier. For example, if the portfolio risk mismatch is arising from the fact that the annuity policies reference an older population (“short longevity” in higher ages) than the life insurance policies, the insurer could go “long longevity” through standardized building-block hedges that reference higher ages, in order to balance the mortality risk profile by age across the book.

**9.7 Conclusions**

In this chapter we have addressed step five in the LifeMetrics Framework by providing a perspective on the hedging of longevity and mortality risk. Deciding on a course of action, whether it be to hedge or retain the longevity and mortality risk associated with an exposure, requires an assessment of hedge effectiveness, along with an analysis of the costs and benefits of hedging. The challenge for risk managers is to structure their hedges so as to minimize the impact of population basis risk. Population basis risk cannot be eliminated, but it can be minimized and managed through the careful construction and calibration of hedges. We have described a standardized building-block approach to hedging longevity and mortality risk that combines the advantages of standardization (liquidity, fungibility, cost, etc.) together with the flexibility needed to design effective hedges for different types of exposures.
This discussion is intended to be introductory and has only just scratched the surface in terms of the measurement and management of longevity and mortality risk. LifeMetrics as a platform for longevity and mortality risk management will continue to evolve and expand as new tools, new methods and new data come to light and find practical application.
Appendix: Technical description of LifeMetrics Index

A.1 Introduction

The construction of the components of the LifeMetrics Index involves a number of methodologies for the transformation of the raw data, as briefly described in Chapter 5. Here, we provide background and explanation of these methodologies.

As discussed in the main text, we believe that transparency is vital to the success of the index. We provide a full technical description of the index calculation in Section A.4 to enable the user to re-construct the index from raw data in a manner consistent with our own implementation.

A.2 Graduation of mortality rates

Graduation is the process by which age-specific mortality rates are smoothed across ages (and sometimes across time) to eliminate statistical noise and data errors in order to ascertain the true underlying trend in mortality. In practice, smoothed mortality data is used to construct the life tables applied in pricing, valuations, and projections by actuaries for both insurance products and pension funds.

The benefits of using graduated mortality rates are that they avoid the errors and inconsistencies which are common in raw data. In particular, graduation is used to aid monotonicity (i.e., increasing mortality rate at increasing age) beyond the accident hump and to reduce the effect of data error (e.g., incorrectly captured number of deaths or exposures) which can result in a mortality rate that is completely out of line with those of the ages nearby. By graduating raw mortality rates, the intention is to identify the “true” underlying mortality rate from which the raw data have arisen. The choice of graduation technique and implementation represent a balance between the desired level of smoothness in the resultant rates and closeness of fit to the raw mortality data.

Graduation techniques can be separated into parametric and nonparametric models. The former define a mathematical model for mortality and the latter smooth the underlying data directly from original data without having to rely on an assumption of an age dependant function. Nonparametric models have the advantage that they eliminate the possibility of incorrectly choosing the underlying model, as they may be applied to any type of data set, whereas an appropriate class of parametric models has to be specifically chosen for mortality rate smoothing.

For the construction of the index, we require a graduation process that is objective, consistent and transparent. These requirements remove any opportunity for subjective intervention which would result in “unpredictable” variation in the set of smoothed rates. If this were not the case, then a user could not be sure that for a given set of data, graduation would follow a specified algorithm.

This is a fundamentally different approach compared with usual practice in graduation (both current and historical) where the objective has been to obtain a reasonable smoothed curve with less regard for consistency of precise algorithm from year to year. That is, there has always been the opportunity to revise parameters, constraints or implementation of the graduation methodology if it were felt that the fitted curve would otherwise not meet with approval in a given year.

We review the choice of methodology for index graduation with a discussion of common methodologies.

A.2.1 Parametric methods

Parametric models make a base assumption on the functional form of the relationship between age and mortality. Once this form has been established, the functional parameters are determined through a process of optimization. This optimization attempts to reduce the difference between the original raw mortality data and the estimated values as determined by the parametric formula. This is typically a numerical rather than an analytic process, usually based on optimizing the sum of squared differences between the actual and estimated data across ages (for instance, in terms of number of deaths), minimizing a chi-square statistic, or through maximum likelihood estimation of the parameters.
A.2.1.1 Gompertz-Makeham (GM)

The most frequently used parametric model is Gompertz-Makeham. The Gompertz part of the model is based on the biological concept of organism senescence, in which mortality rates increase exponentially with age. The Gompertz-Makeham (GM) formula of order \((r,s)\) can be defined by

\[
GM_{r,s}^x = \sum_{i=1}^{\alpha} \alpha_i x^{i-1} + \exp \left[ \sum_{i=1}^{\alpha} \alpha_i x^{i-1} \right]
\]

Mortality rates can be interpreted as a function of an \(r\)-order polynomial and an exponential of an \(s\)-order polynomial in terms of age, \(x\), or even a function of age. The alphas represent a vector of parameters which are determined through the fitting process.

GM models have many good qualities for graduation of mortality data, and are used by the UK’s Continuous Mortality Investigation Bureau (CMIB) to produce mortality tables based on data obtained from UK life offices pensioners’ and annuitants’ experience on a periodic basis. The choice of the correct order of the parameters \(r\) and \(s\) is crucial when the GM formula is used for graduation, as different graduated rates will result from different values of \(r\) and \(s\). Moreover, the order of the parameters that should be used can be different for different mortality experiences. For example, in the construction of the final “00” series mortality tables based on 1999-2002 mortality experience in the UK, the CMIB used GM(0,3), GM(0,4), GM(0,5), GM(1,2), GM(1,3), GM(1,4) and GM(2,2) models for different series (see CMI Mortality Committee Working Papers 21 and 22 (2006) for further details).

After deciding on the appropriate order of the Gompertz-Makeham formula, numerical optimization is then used to find the best fit in terms of a given set of parameters. Although the GM model has been widely used in the UK by the actuarial profession for many years, its application is still challenging, compared with, for example, non-parametric models like kernel smoothing or cubic spline smoothing. These latter methods have the advantage of being easily applied with the use of built-in routines in many mathematical software packages. This could be considered to make them more user friendly to apply and more objective and transparent in that users of the same package will achieve identical results.

A.2.2 Non-parametric methods

Non-parametric graduation of mortality data is carried out by smoothing crude mortality rates obtained directly from the original data. These methods (which include “osculatory” methods) develop smoothed rates by weighting averages of crude mortality rates at neighboring ages. Methods differ in the number of points used in the averaging process and the methodology used to establish the weightings. The methodology typically establishes the weightings by creating a set of curves which are joined at data-points called “knots”. At these points there is continuity of value, and generally smoothness, accomplished by having one or more derivatives match at the knot. Methodologies differ in the number of derivatives that are matched and in the case that the derivative is approximated by numerical computation of the underlying data, by the calculation method (e.g., forwards, backwards or central difference).

In a number of applications, the consequent weightings can be pre-specified in a linear compound form. It is uniquely determined and not based on any data set. Methods often refer to successive “points” that represent groups of data. Grouping by age (e.g., 5 year intervals) is often used to eliminate problems in the data such as digit preference.

Methods that use these data-independent weightings such as Whittaker-Henderson or Beers’ method are certainly transparent. They have remained popular in the US since their introduction early last century. Initially, computing power was limited to the extent that these formulae stretched the operational capabilities of the time. Since then, the ability to compute more sophisticated techniques has increased exponentially, allowing the consideration of more flexible models that can determine the appropriate weights for averaging without pre-determination.

The most popular methodology in which the weights are calculated dynamically depending on the underlying data is that of spline smoothing. The GAD in the UK has used cubic spline smoothing in various forms for the production of the last few decennial English Life Tables. Subject to a decision
on appropriate level for a smoothing parameter, which we discuss below, cubic spline smoothing can be considered objective and transparent, especially given the familiarity of many users with cubic splines from other areas of financial application. For index calculation, we reject more sophisticated approaches such as locally-weighted regression and Generalized Additive Models (Debon et al., 2006) on the basis that they are complex and the lay user may not find them as transparent or simple to implement.

A.2.2.1 Beers’ ordinary minimized fifth difference method

Beers’ method is in common usage in the US, applied by both the SSA and the CDC to construct their annual mortality tables by single year of age. These agencies follow the process of aggregating the underlying deaths/exposures into 5-year age intervals. Then, values for the number of deaths and population by single year of age are obtained by Beers’ graduation. Once the graduated values of deaths and populations have been determined, ratios are taken to calculate the mortality rates; i.e., no smoothing is applied to the mortality rates themselves.

For details of how the linear compound form for an oscillatory graduation method can be derived, we refer the reader to the derivation of Beers’ interpolation formula, as set out in the Record of the American Institute of Actuaries, November 1944.

A.2.2.2 Spline smoothing

A spline is a piecewise polynomial function which incorporates certain assumptions regarding how the pieces are joined together at the knots. Splines are distinguishable from other types of piecewise polynomial functions in that at each knot the spline is differentiable some defined number of times (specifically the degree of the polynomial minus one). A spline is therefore defined by the locations of the knots and the coefficients of the polynomials which define the functions on each interval.

Spline smoothing determines the coefficients of the polynomials in order to minimize the function

\[ \sum_{j=1}^{n} (m_j - f(x_j))^2 + \lambda \int_{x_k}^{x_{k+1}} (f^{(4)}(t))^2 \, dt \]

where the knots are located at \( x_1, x_2, \ldots, x_n \).

The first term here represents goodness of fit by means of mean squared differences between the graduated data, \( f(x_j) \), and raw data, \( m_j \). The second term is a penalty term which increases with fluctuations of the polynomial. The smoothing parameter, \( \lambda \), represents the tradeoff between smoothness (here, in terms of change in the \( k \)th derivative) and goodness of fit (i.e., residual errors between the graduated and raw data). A higher lambda indicates there is a greater penalty for lack of smoothing.

When \( k=2 \), i.e., we penalize smoothness in terms of the second derivative, it can be shown that a cubic spline where there are knots at every age \( x \) is the unique solution to the minimization (Reinsch, 1967). The cubic spline can be expressed as

\[ f(x) = a_j(x - x_j)^3 + b_j(x - x_j)^2 + c_j(x - x_j) + d_j \]

on each interval \([x_j, x_{j+1}]\) for \( j=1,\ldots,n-1 \).

We note that it is possible to weight the contribution of the term for each age in the optimization. For instance, weighting by a function related to the variance of the mortality rate at each age would increase the influence of more reliable data points on the overall shape of the graduated curve. Alternatively, one can choose a suitable transformation to apply to the raw data before graduation which makes the level of transformed mortality at each age more comparable.

In order to carry out the optimization, one can create a set of non-singular linear equations. In matrix form, this translates to a tri-diagonal problem which can be solved via Cholesky decomposition. We refer the reader to Reinsch (1967) for a full description of this methodology.

The UK Government Actuary’s Department (GAD) moved from parametric methods to cubic splines in the construction of the English Life Tables following experimentation versus parametric methods by McCutcheon and Eilbeck in the 1970’s.
In the most recent series, based on data from 1990-1992 (English Life Tables No. 15), the GAD avoided the requirement to determine the number and position of the knots by defining a knot at each age.

A.2.3 Testing graduation techniques

Graduation tests review whether the deviations between the original data and the graduated data are randomly distributed, independent, and distributed in accordance with the assumptions inherent in the underlying model. We outline a number of tests which can be considered for the evaluation of smoothed data.

A.2.3.1 Signs test

This test compares the raw data to the graduated rates, and at each age, determines whether the sign of the deviation is positive or negative. The test identifies the number of positive and negative deviations and determines whether there is an imbalance between them. Under the assumption that the deviations are random and independent, the number of positive or negative signs should be binomially distributed with both signs being equally likely to occur.

A.2.3.2 Runs test (Grouping of signs)

This test looks at the number of groups of deviations of the same sign and compares this with the number that would be expected if the positive and negative signs were arranged in a truly random order. If the graduation technique is appropriate, then the deviations at successive ages should be independent, signs should be randomly distributed, and there should be neither too few nor too many runs of successive deviations with the same sign.

A.2.3.3 Chi-square goodness of fit

This test examines how closely the graduation fits the original data. For each age \( x \), calculate the standardized deviation \( z_x \), by:

\[
z_x = \frac{d_x - E_x m_x}{\sqrt{E_x m_x}}
\]

where \( d_x \) is the actual number of deaths at age \( x \), \( E_x \) is the actual exposed to risk at age \( x \), and \( m_x \) is the graduated rate of mortality at age \( x \).

The test statistic, \( \sum z_x^2 \), is the sum of squared standardized deviations over all ages under consideration and is compared with a chi-square distribution with degrees of freedom equal to the number of age groups covered minus the number of restrictions implied by the model used to graduate the data.

One must bear in mind that the chi-square test can indicate the overall goodness of fit but will fail to detect certain defects that other tests would pick up. This means that the chi-square test by itself is not sufficient to indicate if the data is well-graduated.

A.2.3.4 Serial correlations

If data is well-graduated, one should expect that the standardized deviations, described above, would be independent. This test analyzes the correlation between standardized deviations at neighboring ages, where “neighbors” are determined by a lag \( j \). The test statistic is tested against the standard normal distribution under the assumption that the standardized deviations are independent.

A.2.3.5 Standardized deviations

This test examines the distribution of the values of standardized deviations. They should be an independent sample from the standard normal distribution. To perform this test, the number line is split into intervals and the observed number of \( z_x \) in each interval is evaluated. This is compared with what should be expected under a standard normal distribution using a chi-square test.

A.2.3.6 Cumulative deviations

The cumulative deviations test detects overall bias or long runs of deviations of the same sign. It is based on the hypothesis that

\[
d_x \sim \text{Normal}(E_x m_x, E_x m_x)
\]

where \( d_x \) is the actual number of deaths at age \( x \), \( E_x \) is the actual exposed to risk at age \( x \), and \( m_x \) is the graduated rate of mortality at age \( x \).
The test statistic,
\[ T = \frac{\sum (d_x - E_m_x)}{\sqrt{\sum E_m_x}} \]
is compared with a standard normal distribution as a two-tailed test.

A.2.4 Index graduation testing

In order to identify the most appropriate graduation methodology for the LifeMetrics Index, we test the models previously highlighted as the most suitable potential candidates. For this purpose, we test:

- Beers’ ordinary minimized fifth difference method
- Cubic spline smoothing

For the implementation of cubic spline smoothing, we use the formulation of Schoenberg and Reinsch, measuring smoothing in terms of the second derivative. In terms of the raw data, \( m_x \) and the desired smoothed output, \( s_x \), for a given smoothing parameter \( p \) in the range \([0, 1]\), we minimize

\[ p \sum_x (m_x - s_x)^2 + (1 - p) \int \frac{d^2 s_x}{dx^2} \]

Given that the solution can be shown theoretically to be minimal for a cubic spline with a knot at every age in the region to be smoothed, we follow this choice of knots. (For the reader wondering why we have not mentioned P-Splines as a potential smoothing methodology, we note that the cubic spline methodology is in fact the equivalent of P-Splines when there are knots at every age and these are equidistant. For further details, see the comments by Engel and Kneip in “Flexible Smoothing with B-Splines and penalties”, Eilers and Marx, 1996).

For an objective implementation, we need to avoid the necessity to choose \( p \) with respect to the particular data for a given year. We could consider the implementation of a methodology which identifies an optimal \( p \) as part of the process (for instance, by cross validation). However, motivated by a greater need for simplicity and transparency, we instead choose to fix the level of \( p \). For the same reasons, we avoid weighting the individual terms in the summations, by variance for example. We note that an effective transformation of the underlying variable (such as using \( \log(m_x) \)) reduces the requirement for such a weighting. We test the graduation for a number of fixed values for the smoothing parameter.

We test both methodologies on United States data from 1968 to 2003. In addition, we apply the cubic spline smoothing algorithm to England & Wales data for 1971 to 2004. Criteria considered are suitability in terms of shape and smoothness of the graduated mortality rates relative to the raw data. (For instance, we should generally see a monotonically increasing mortality rate beyond a certain age). In addition, we analyze the smoothed rates with a battery of tests, as described in the previous section, considering results for each year for:

- Signs test
- Runs test
- Serial correlations
- Standardized deviations
- Cumulative deviations

Both methodologies are tested for graduation of raw \( m_x \) data and also for graduation of the transformed raw data in the form \( \log(m_x) \). The latter is well known in the industry as a transformation of the shape of mortality rates to be more linear with age, for which there is evidence that graduation may be more effective (Debon et al., 2006).

In addition, we consider graduation of the initial rate of mortality in the form \( q_x \) and \( \log(q_x) \) (after calculating the values \( q_x \) using the approximation described in 5.2.2) and find that, in practice, graduating \( q_x \) or \( \log(q_x) \) has no beneficial effect as compared to graduating \( m_x \) and then calculating the corresponding values of \( q_x \).

A.2.4.1 Test results

On the United States dataset, we find that Beers’ methodology passes the signs, runs, serial correlations and cumulative deviations tests for males for all reference years between 1980 and 2003 at a 5% significance level. However, between 1968 and 1979, there are a number of years for which either or both of the serial correlations and cumula-
tive deviations tests fail. Mortality rates are monotonically increasing beyond age 35 in all years. For females, the situation is similar other than a single instance in 2003 when the signs test is rejected. However, other than a couple of years in which the graduation for females passes, the graded data from Beers’ methodology consistently fails the standardized deviations test for both males and females. Results are similar at a 1% significance level.

For the cubic spline smoothing, we note that there is a trade-off between fit and smoothness based on the choice of value for the smoothing parameter, $p$. For low levels of $p$, smoothing is stronger at the expense of closeness of fit to the raw data. We therefore expect to find that at the extremes, the methodology will start to fail the graduation tests. Indeed this is the case and for $p$ in the region of 0.2 and below, we begin to fail the signs test for some years. In similar fashion, the proportion of years for which we fail the standardized deviations test increases as $p$ decreases. However, for high levels of $p$ (above 0.8), there are years for which the graduated rates are not entirely monotonic beyond age 35. For values of $p$ between 0.2 and 0.8, the graduation passes the signs, runs, serial correlation and cumulative deviation tests at a 5% significance level for both males and females. So, for a suitable choice of $p$, the cubic spline provides better results than those obtained using the Beers’ method for both males and females. We note that for both methods, the standardized deviations test is not consistently passed. We propose one possible reason for this as the high level of noise associated with the US data.

We note that when $p$ is within the range 0.2 to 0.8, the results for males are improved when the graduation is applied on $\log(m_x)$ rather than directly on the raw $m_x$. For females there is no clear additional benefit of graduating the $\log(m_x)$ rather than directly graduating $m_x$.

On the England & Wales dataset, we focus on the use of the cubic spline graduation method, given its superiority on the US data. We find that cubic spline smoothing is effective for a wide range of $p$ values, passing the signs, runs, serial correlations and cumulative deviations tests in all years between 1971 and 2004 for both males and females at 5% significance level. However, for higher values of $p$, in particular beyond 0.4, where there is less smoothing, the standardized deviations test does not perform as well. These results are consistently observed over time for both males and females and at 1% and 5% confidence levels. In addition, for $p$ above 0.6, we start to see years in which graduated rates are non-monotonic above age 35, especially for the male population. Results are improved slightly for both males and females when the graduation is applied on $\log(m_x)$ rather than directly on the raw $m_x$. So in the case of the EW data, a value between 0.2 and 0.4 seems to be a reasonable choice for the smoothing parameter $p$.

### A.2.5 Choice of graduation methodology for index calculation

Following testing, we select the cubic smoothing spline algorithm as the graduation methodology for index calculation. We graduate the logarithm of the raw central rates of mortality, with a fixed value for the smoothing parameter, $p$, of 0.375 (which is within the range of values suggested by the results we have obtained for both US and EW data). This is consistent with anecdotal evidence from Mathworks that satisfactory smoothing is often obtained for a value of

$$p = \left(1 + \frac{h^2}{0.6}\right)^{-1}$$

where $h$ is the distance between knots (here, 1 year). The Mathworks, Inc. are the developers of the technical computing language Matlab.

We use Matlab for the calculations (See [http://www.mathworks.com/products/matlab/](http://www.mathworks.com/products/matlab/) for further details). For cubic spline smoothing, we use the csaps.m routine which is based on the Fortran routine SMOOTH from PGS, as described in de Boor (2001). csaps.m returns the cubic smoothing spline in “ppform” (tensor product form). To extract the value of the graduated mortality rates for each age, we use the standard Matlab routine, ppval.m. A process map for the calculation is shown in Figure A.1.

This methodology creates a graduated set of mortality rates for the range of the raw data (up to age 84 in the United States, and 89 in England & Wales). For the Index, we require publication of all individual age mortality rates up to age 90. We therefore need to calculate mortality rates for a small number of additional ages (85-90 inclusive in the United States, 90 in England & Wales). To this end, we extrapolate the data in a manner consistent with the gradua-
tion. This is a very transparent process which is valid over a small interval such as those considered. Over longer age ranges, it would result in unrealistic assumptions (and unbounded mortality rates).

The extrapolation of the graduated data is a linear extrapolation which matches the level and slope of the (continuous) logarithm of the mortality rate at the final age (i.e., the final smoothing spline). In implementation, we use the fnxtr.m routine from Matlab for this purpose. A process map for the calculation is shown in Figure A.2.

### A.3 Methods for estimating mortality rates at higher ages

The estimation of population and mortality data at high ages has always been subject to significantly increased error. There are a number of reasons why this has been the case:

1. For the very old, birth records may be unavailable and individuals may not know their true age for accurate census reporting
2. An individual may also knowingly inflate his age. For example, there can be a tendency to report an age of 100 or over for persons of advanced age
3. For death data, it is frequently the case that age at death is unknown, or is inaccurately estimated by family members
4. At high ages, despite increased mortality rates, the lower population size means that the absolute number of deaths in a year is typically small. This magnifies the impact of noise in the data

Each of these issues contributes to a lack of credibility for raw older age mortality data. Based on life tables published by national governmental agencies, it appears that each has taken a view on the last age for credible data. For instance, in the United States, it could be concluded that data appears to be credible up to age 84 inclusive (Siegel & Swanson, 2004). After age 84, the Centers for Disease Control and Prevention (CDC) publish mortality rates based on experience with reference to a secondary source, as described in ‘National Vital Statistics Report, Vol. 54, No. 14. 2006’. In England & Wales, national data is currently published by single year of age only up to age 89 inclusive, and the CMIB have taken a similar approach to cut-off (Forfar et al., 1988).

In order to properly calculate life expectancy, mortality rates are required for each age until the mortality table is closed out with a mortality rate of 100%. For example, a common assumption is to close out the table with a mortality rate of 100% at age 120. For determining mortality up to this age, appropriate methods of developing mortality rates at higher ages based on the raw data must be considered.

There are a number of well known methodologies with differing complexities. At one end of the spectrum, a popular approach is to fit a parametric form to the high age mortality rates and calibrate this based on a combination of raw data and desired shape. More advanced methods which can be appropriate given sufficient quality of underlying data are the method of Extinct Generations, and further developments of this - the Survivor Ratio method and the Das Gupta Advanced method. These are particularly relevant when data on deaths is considered to be wholly reliable but there is difficulty with the accurate estimation of population exposures at the higher ages.

Figure A.1 Index graduation process map
A.3.1 Parametric approach

There is strong evidence in mortality data that the rate of change of mortality reduces at high ages. Parametric forms (such as Gompertz-Makeham) used at lower ages can overestimate these mortality rates and consequently, other actuarial mortality models can be more suitable in shape. For instance, the Kannisto formula projects for mortality rates to plateau at high ages, and has the form:

\[ m(x) = \frac{ab^f(x)}{1 + ab^f(x)} \]

where \( f(x) \) is a linear function of age (Thatcher et al., 1998).

Alternatively, a simple polynomial form can be fitted to raw data such that the mortality rate (and/or derivatives) match at specified data points which are still considered to be reliable (for instance, mortality rate at age 89). Any remaining degrees of freedom can then be used fit to a desired shape or specific condition (e.g., value) at higher ages. The English Life Tables in recent years have used cubic polynomials fitted to \( m_x \) (ELT 14) and linear regression on logit \( q_x \) (ELT 15) for this purpose (Haberman, 1997).

A.3.2 Method of Extinct Generations

The method of Extinct Generations, also known as the method of Extinct Cohorts, was first proposed by Vincent (1951). This method serves to produce more reliable estimates of populations at higher ages than census populations. Once these population estimates are determined, more accurate mortality rates may be determined by combining these estimates with death data (which is assumed to be reliable).

Under the method of Extinct Generations, once all the members of a cohort have died, dates of death and dates of birth for each member of the cohort can be used to determine the number of individuals alive in the cohort at previous times. This can be used to reconstruct accurate populations for cohorts in earlier years, although this is only possible for cohorts for which no living individuals remain.

There are two main assumptions behind the method of extinct cohorts. One is that death registration data is sufficiently accurate for the purpose (and, in particular, more accurate than census population data). The second assumption is that net migration at these ages is negligible.

A.3.3 Survivor Ratio method

The Survivor Ratio method is an extension of the previous methodology to calculate population estimates for cohorts that are almost extinct. The idea behind this is that once a cohort is sufficiently aged, e.g., 100 years old, only a small number of the original cohort members will be surviving. The ratio of this small number of survivors to the number of deaths over the past, say, five years, can be definitely determined from previous (extinct) cohorts. This is the so-called “survivor ratio.”

After calculating such a survivor ratio at age 100 for an extinct cohort, the ratio (or a slightly adjusted ratio) may be multiplied by the actual number of deaths over the prior five years for an almost extinct cohort to estimate the number of survivors in the almost extinct cohort at age 100. Working backwards, the number of survivors at earlier years can then be determined by adding this estimated number of survivors to the actual recorded deaths for previous years.

A.3.4 Das Gupta Advanced method

The Das Gupta Advanced (DA) method as proposed by Andreev (2004) is similar to the Extinct Cohort method and the Survivor Ratio method. Where the Survivor Ratio method
calculates survivor ratios, the DA method calculates “death ratios,” which are the n-year ratios of [deaths at age x+1] to [deaths at age x] for a prior cohort. The DA method combines these death ratios with population data at younger ages to produce indirect estimates of mortality rates for the current year. The younger age population data is essentially used to calibrate parameters which feed into the estimation of population at the older ages.

A.3.5 Using alternate data sources

A fundamentally different approach to higher age mortality is to utilize alternate data sources where they are available and relevant. For instance, in the US, the CDC uses Medicare data for calculating mortality rates above the age of 84. Pooled Medicare data averaged over, say, a three year period is used to calculate mortality rates for ages 85-99. These mortality rates are then extrapolated for ages 100 and above (Vital and Health Statistics, No 129).

A.3.6 Choice of high ages fitting methodology for index calculation

We require mortality rates beyond age 90 in order to perform our calculations of life expectancy for the index. We do not explicitly publish the mortality rates themselves at these ages and note that the sensitivity of the published figures for life expectancy to these mortality rates is still relatively low.

We opt for simplicity and transparency in our choice of high ages methodology, and choose to fit a parametric form to the data. Since data for deaths is not considered wholly accurate at these ages, the more complex methodologies that make this underlying assumption are not considered appropriate. We plan to continue our research into high ages methodology with a view to developing a more sophisticated algorithm that is appropriate for the available data and this may be used to update life expectancy calculations in future index releases.

Our current methodology is to fit a functional form that provides a sensible approximation to the path that mortality rates would be expected to follow at high ages. For consistency with the functional form used in the graduation algorithm, we use a cubic polynomial for this purpose. The cubic polynomial is used to model the central mortality rates, m. We note that if we were to model the logarithm of the central mortality rates, the cubic polynomial would not be a suitable form and another function would need to be used. This is to avoid an increase in the potential that the polynomial reaches an inflection point and local maximum, creating a non-monotonic shape.

The fitting process calibrates the polynomial to match the level and continuous first derivative of the mortality rate curve at age 90. Further constraints are applied at age 119 as follows:

1. Value at age 119 is set to a fixed level
2. First derivative of value at age 119 is set to zero

The fixed level at age 119 is set with reference to initial rates of mortality in standard tables for the geography in question. We fix the level such that the equivalent initial rate of mortality, q, is equal to 50% in the United States and 60% in England & Wales (consistent with, for instance, RP-2000 and 1994 GAM for the US, PCMA00 and ELT15 for EW).

The mortality rate at age 120 is then set equal to 100% to close the mortality table. Note that closing the table at age 120 is common practice in mortality table creation (See, for instance, CMIB working papers 21 and 22).

The polynomial fitting process is accomplished through algebraic manipulation of the four constraints on level and first derivative at the two endpoints (90 and 119). We implement the solution of this problem through matrix inversion in Matlab. A process map for the calculation is shown in Figure A.3.

A.4 Index technical description

For a given year, t, we begin with raw data for deaths and population obtained from the relevant national government agencies. This data consists of:

- Actual number of deaths in the year, for each age x: \( D_{x,t} \)
- Mid-year population estimate in the year, for each age x: \( E_{x,t} \)
A.4.1 Crude central rate of mortality, $m_x$

The central rate of mortality at age $x$ is calculated as the number of deaths in the year for age $x$ divided by the mid-year population estimate in the year for age $x$, i.e.:

$$m_x = \frac{D_x}{E_{x+x}}$$

In the LifeMetrics Index, we publish the crude central rate of mortality as a percentage, to 4 decimal places. For England & Wales since 1971, we publish the rates for ages 20 to 89 inclusive. From 1961 to 1970 the published rates cover the age range 20 to 84 inclusive. For the United States we publish the rates for ages 20 to 84 inclusive. Beyond these ages, raw data by individual age is currently unavailable or considered to be unreliable due to difficulty in recording high age data accurately.

A.4.2 Graduated initial rate of mortality, $q_x$

The initial rate of mortality is calculated from the set of crude central mortality rates for a given year. First, we graduate the central mortality rates, $m_x$, to produce smoothed central mortality rates, $s_x$, by minimizing

$$p \sum_i (\log(m_i) - \log(s_i))^2 + (1 - p) \left( \int \frac{d^2 \log(s_i)}{dx^2} \right)^2$$

where $p = 0.375$, and $\log(s_i)$ is constructed as a set of cubic splines, with a different spline between every two ages in the crude data, and continuity of slope at each age.

For England & Wales since 1971, we begin with raw data for the age range 20-89 inclusive and produce graduated rates for 20-89 inclusive. From 1961 to 1970, we begin with raw data for the age range 20-84 inclusive and produce graduated rates for 20-84 inclusive. For the United States, we begin with raw data for ages 20-84 inclusive and produce graduated rates for 20-84 inclusive. Beyond these ages, raw data by individual age is currently unavailable or considered to be unreliable due to difficulty in recording high age data accurately.

A.4.3 Period life expectancy, $e_x$

The calculation of period life expectancy requires a full set of initial mortality rates up to a final age at which the mortality rate is 100% (assumed here to be at age 120). Given this set of rates, we calculate period life expectancy using the formula

$$e_x = \sum_{i=1}^{N} \left( \prod_{j=1}^{x-i} (1 - q_{x-j}) \right)$$

For initial mortality rates up to age 90, we use the rates as published in the index for the year (and as described in the previous section). Beyond age 90, where we have not explicitly calculated a mortality rate, we calculate the mortality rates by fitting a single cubic polynomial of the form

$$f(x) = ax^3 + bx^2 + cx + d$$

to calculate the central rate of mortality, where $x$ represents age, between ages 91 and 118. (We do not directly publish mortality rates beyond age 90 since there is a fair degree of uncertainty in their estimation. We note that the impact these rates have on uncertainty of life expectancy is far less significant.)

The parameters $a$, $b$, $c$, $d$ are calibrated by the solution of the set of linear equations:

- $f(90) = \text{value of graduated central mortality rate curve at age 90 (known)}$
- $f'(90) = \text{slope of graduated central mortality rate curve at age 90 (known)}$
- $f(119) = \text{defined level (see below)}$
- $f'(119) = 0$

where $'$ represents first derivative with respect to age, i.e., slope.
The fixed level at age 119 is set with reference to observed rates of mortality in standard tables for the geography in question. We use the equivalent central mortality rate of an initial mortality rate of 50% in the United States, or 60% in England & Wales (consistent with, for instance, RP-2000 and 1994 GAM for the US, PCMA00 and ELT15 for EW). I.e., central mortality rates of 66.67% and 85.71% respectively.

Having determined the central rate of mortality from the fitted polynomial, we then transform the results using

$$q_x = \frac{f(x)}{1 + \frac{1}{2}f(x)}$$

to determine the initial rates of mortality between ages 91 and 118. Finally the initial mortality rate at age 120 is set equal to 100% to close the mortality table.

In the LifeMetrics Index, we publish period life expectancy in years, to two decimal places, for ages from 20 to 80 inclusive.

A.4.4 Dealing with known mortality effects in the data

The “cohort effect” in England & Wales is a well documented phenomenon in which certain cohorts of the population, principally those born around 1930, have demonstrated consistently stronger mortality rate improvements over time than those cohorts born near them. In addition to this, there are a small number of cohorts for which mortality rates themselves have been markedly out of line with those of the cohorts around them. Principally, there are two cohorts of interest for which mortality rates are noticeably lower than their neighbors:

- The cohort born in 1886. There is a supposition that this may be down to mis-stated population exposures (see Renshaw & Haberman (2006), for example), and this is backed up by the fact that mortality rates are back in line with neighbouring cohorts in 1971 after the census update of

![Figure A.4 Crude central mortality rate for England & Wales, Males and Females, 1969](image)
the population size. Figure A.4, illustrates this cohort effect for males and females in 1969 (note: the data points for the second highest age).

- The cohort born in 1919. This generation were born during the 1919 influenza epidemic and at the end of the first World War. The combination of these two events are supposed to have influenced subsequent mortality rates (see Richards (2007)).

The graduation of mortality rates smoothes out the effect of these data points in index calculation apart from in the instance that the cohort happens to be the final raw data point available. In this case, the cubic smoothing spline algorithm will make the assumption that the downturn in mortality rate is an ongoing phenomenon and subsequent extrapolation for higher ages will continue the downwards trend, an undesirable prospect given that we know the effect is contained at the one age.

Consequently, when an identified cohort with markedly lower mortality reaches the age at which it becomes the final data point for graduation, it is excluded for the calculation of graduated initial rates of mortality. Instead, the smoothing spline graduation algorithm is terminated at the previous data point. Extrapolation will then be applied from this lower age up to age 90 inclusive.

In the index calculation, we have used this methodology for the construction of graduated rates in reference year 1970 (when the 1886 cohort reached age 84) for England & Wales. We envisage that the same action will be necessary in reference year 2008 when the 1919 cohort reaches age 89, for England & Wales.

The final decision to take this action will be the remit of the Index Advisory Committee and notified publicly well in advance of publication of the index values. The Advisory Committee will be responsible for making similar decisions on new cohorts going forwards but we note that in our survey of the academic and practitioner literature, the two examples described above are at present unique.
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